Square root
and
Cube root

Topic Overview – Square root and cube root

- Square and Cube of numbers
- Square root and cube root of a number
- Methods to find Square root
  - → Prime factorization method
  - → Division method
- Approximate value of a square root
- Square root of a fraction
- Square root of a decimal
- Find cube root of a number
- Illustrations

Square of a number

- When a number 'n' is multiplied by itself, the product is called Square of number 'n'.
- Square of n is written as n^2. $\therefore n^2 = n \times n$

Example: (i) $4^2 = 4 \times 4 = 16$

(ii) $(-6)^2 = (-6) \times (-6) = 36$

(iii) $\left(\frac{4}{3}\right)^2 = \left(\frac{4 \times 4}{3 \times 3}\right) = \frac{16}{9}$

Table of Squares of numbers

Number	Square	Number	Square	Number	Square
1	**1**	11	**121**	21	**441**
2	**4**	12	**144**	22	**484**
3	**9**	13	**169**	23	**529**
4	**16**	14	**196**	24	**576**
5	**25**	15	**225**	25	**625**
6	**36**	16	**256**	26	**676**
7	**49**	17	**289**	27	**729**
8	**64**	18	**324**	28	**784**
9	**81**	19	**361**	29	**841**
10	**100**	20	**400**	30	**900**

→ **Memorize squares of numbers up to at least 30.**

Squares are interesting numbers

Just see the pattern below….

$1^2 = 1$

$2^2 = 4 \quad = 1+3$

$3^2 = 9 \quad = 1+3+5$

$4^2 = 16 \quad = 1+3+5+7$

$5^2 = 25 \quad = 1+3+5+7+9$

$6^2 = 36 \quad = 1+3+5+7+9+11$

$\Rightarrow$ **n^2 = Sum of first n odd numbers**

Interesting !

Cube of a number

- The number obtained when a number ‘n’ is multiplied with itself two times is called cube of ‘n’.
- Cube of n is written as n^3. $\therefore n^3 = n \times n \times n$.

Example: $4^3 = 4\times4\times4 = 64$ $(-5)^3 = (-5)\times(-5)\times(-5) = -125$

Table of cubes of numbers

Number	Cube	Number	Cube
1	**1**	7	**343**
2	**8**	8	**512**
3	**27**	9	**729**
4	**64**	10	**1000**
5	**125**	11	**1331**
6	**216**	12	**1728**

Basic properties of squares & cubes

- Square of an odd number is odd and square of an even number is an even number.
- Square of a number is always positive.
- Cube of an odd number is odd and cube of an even number is even.
- Cube of a negative number will be negative.

Square root of a number

- If $x^2 = y, \Rightarrow \sqrt{y} = x$.
- The symbol $\sqrt{}$ is called radical sign and it denotes square root.
- Example: $\sqrt{49} = 7$, $\sqrt{81} = 9$
- Square root of a negative number is not defined under Real Numbers.

Square root of numbers

- Every whole number has a square root.
- Numbers such as 25, whose square roots are whole numbers are called Perfect squares.
- Most numbers are not perfect squares, so their square roots are not whole numbers but irrational numbers.

Methods for finding Square root

Prime factorization method

Steps:	Example
1. Express the given number as the product of prime numbers	$900=2\times2\times3\times3\times5\times5$
2. Choose one out of every pair of same prime number.	2 3 5
3. The product of these gives square root of the number.	$\sqrt{900}=2\times3\times5 = 30$

Example: Find √15, 876 by Prime factorization method.

Solution:

2	15, 876
2	7, 938
3	3, 969
3	1, 323
3	441
3	147
7	49
7	7
	1

$\therefore \ 15,876 = 2^2 \times 3^4 \times 7^2$

$\therefore \sqrt{15,876} = 2 \times 3^2 \times 7 = 126$

Methods for finding Square root

Division method

The given number is first separated into pairs starting from right to left. Each pair of numbers is called a group or a period. The process proceeds one group at a time.

Step-1: The integer whose square is less than or equal to the first group is written in the quotient and divisor place.

Step-2: The square of the integer is subtracted from the first group.

Step-3: The next group is brought down and written with the remainder and forms the new dividend.

Step-4: Double the existing quotient. A new suitable digit is put at the end of this number to get the new divisor. The new digit is selected such that the product of the new digit and the new divisor is the closest to but lower than the dividend. We subtract this product from the dividend. The new digit is put down in the quotient.

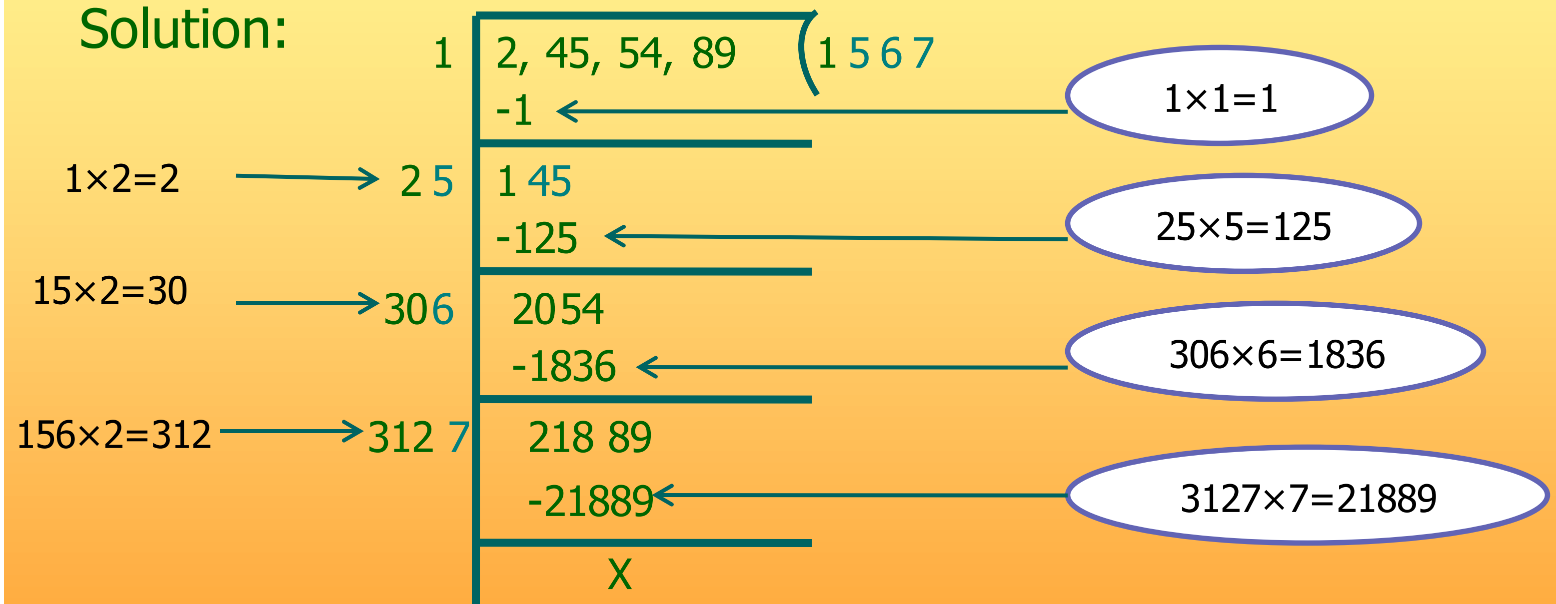
Step-5: We repeat the above two steps till the end of the number.

Let us see an example on this method.

Example: Find the square root of 2455489 using division method.

Make pairs of digits starting from right

Solution:



$\therefore \sqrt{2455489} = 1567$

Approximate value of square root

- A number of times, approximate value of square root is sufficient.
- The basic idea is to find the two consecutive whole numbers that the square root lies between.

Example: Find approximate value of $\sqrt{150}$.

We know that $\sqrt{144} = 12$ and $\sqrt{169} = 13$. $\Rightarrow$ $12 < \sqrt{150} < 13$

Example: Find approximate value of $\sqrt{10}$.

$\sqrt{9} = 3$ and $\sqrt{16} = 4$. $\Rightarrow$ $3 < \sqrt{10} < 4$

Further, $31^2 = 961$ and $32^2 = 1024$

$\therefore$ $3.1^2 = 9.61$ and $3.2^2 = 10.24$ $\Rightarrow$ $3.1 < \sqrt{10} < 3.2$

Square root of a fraction

- Square root of a fraction = square root of numerator ÷ square root of denominator, i.e. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Example: $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

Example: $\sqrt{\frac{294}{486}} = \sqrt{\frac{98}{162}} = \sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$

Square root of a decimal number

- First convert the decimal number to fraction and proceed as earlier.

Example: $\sqrt{0.25} = \sqrt{\frac{25}{100}} = \frac{5}{10} = 0.5$

$$\sqrt{0.0025} = \sqrt{\frac{25}{10000}} = \frac{5}{100} = 0.05$$

$$\sqrt{0.000025} = \sqrt{\frac{25}{1000000}} = \frac{5}{1000} = 0.005$$

$$\sqrt{7.84} = \sqrt{\frac{784}{100}} = \frac{28}{10} = 2.8$$

Caution ! Count the number of decimals very carefully.

Square root of a decimal number

Division method

- If the square root is not a rational number, we use division method.
- The method proceeds exactly the same way as for whole numbers except that we pair the digits before and after the decimal separately.
- The digits of integral part are paired from right to left.
- The digits in fractional part are paired from left to right.
- Remember, we can add additional zeros at the end as needed.

Square root of a decimal number

Example: Find the square root of 2.749

Solution:

1	2.74,90,00	1 . 6 5 8
	-1	
2 6	1 74	
	-156	
325	1890	
	-1625	
330 8	265 00	
	-26464	
	36	

∴ **$\sqrt{2.749}$ = 1.658 correct to 3 decimal places.**

Cube Root

We use prime factorization method.

Example: Find cube root of 5832 using prime factorization method.

Solution:

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\therefore \quad 5832 = 2^3 \times 3^6$

$\therefore \sqrt[3]{15,876} = 2 \times 3^2 = 18$

Thanks…