Quantity of Heat

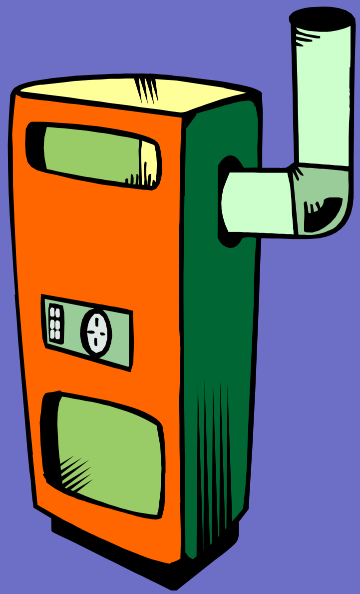
Photo © Vol. 05 Photodisk/Getty

FOUNDRY: It requires about 289 Joules of heat to melt one gram of steel. In this chapter, we will define the quantity of heat to raise the temperature and to change the phase of a substance.

Objectives: After finishing this unit, you should be able to:

- Define the quantity of heat in terms of the calorie, the kilocalorie, the joule, and the Btu.

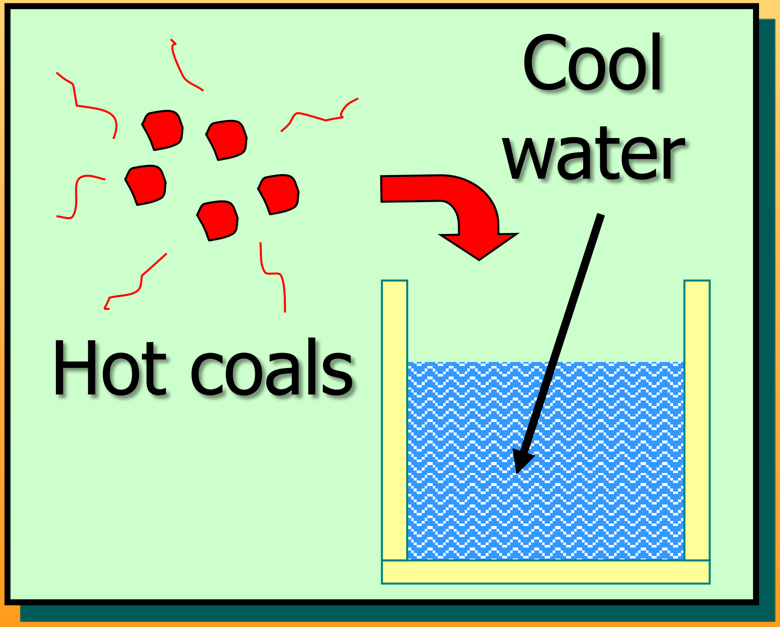
- Write and apply formulas for specific heat capacity and solve for gains and losses of heat.

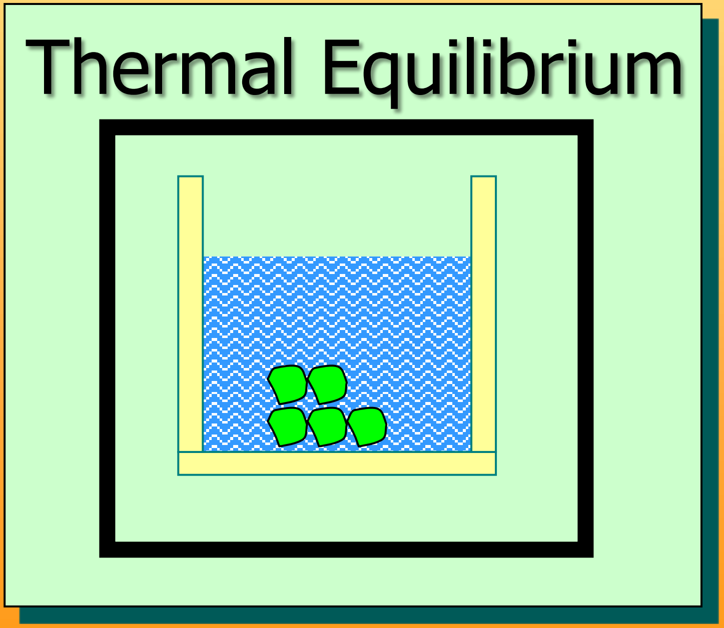


- Write and apply formulas for calculating the latent heats of fusion and vaporization of various materials.

Heat Defined as Energy

Heat is not something an object has, but rather energy that it absorbs or gives up. The heat lost by the hot coals is equal to that gained by the water.





Units of Heat

One calorie (1 cal) is the quantity of heat required to raise the temperature of 1 g of water by 1 C^0.

Example

10 calories of heat will raise the temperature of 10 g of water by 10 C^0.

Units of Heat (Cont.)

One kilocalorie (1 kcal) is the quantity of heat required to raise the temperature of 1 kg of water by 1 C^0.

Example

10 kilocalories of heat will raise the temperature of 10 kg of water by 10 C^0.

Units of Heat (Cont.)

One British Thermal Unit (1 Btu) is the quantity of heat required to raise the temperature of 1 lb of water by 1 F^0.

Example

10 Btu of heat will raise the temperature of 10 lb of water by 10 F^0.

The Btu is an Outdated Unit

The British Thermal Unit (1 Btu) is discouraged, but unfortunately remains in wide-spread use today. If it is to be used, we must recognize that the pound unit is actually a unit of mass, not weight.

1 lb → (1/32) slug

When working with the Btu, we must recall that the pound-mass is not a variable quantity that depends on gravity --

one reason that the use of the Btu is discouraged!

The SI Unit of Heat

Since heat is energy, the joule is the preferred unit. Then, mechanical energy and heat are measured in the same fundamental unit.

Comparisons of Heat Units:

1 cal = 4.186 J

1 Btu = 778 ft lb

1 kcal = 4186 J

1 Btu = 252 cal

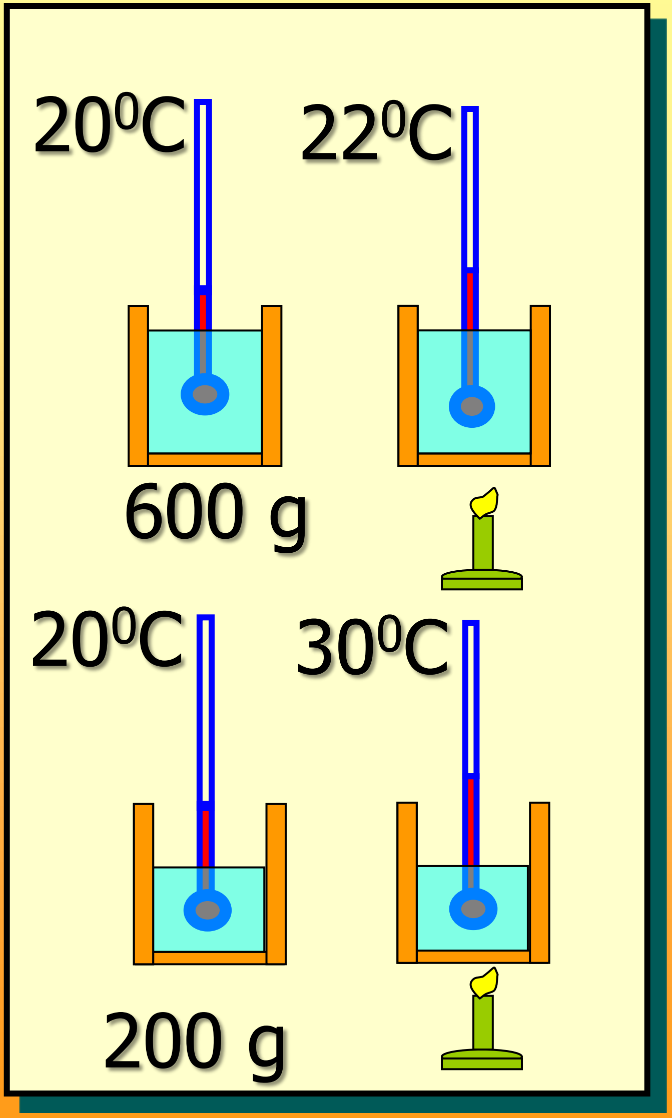
1 Btu = 1055 J

Temperature and Quantity of Heat

The effect of heat on temp- erature depends on the quantity of matter heated.

The same quantity of heat is applied to each mass of water in the figure.

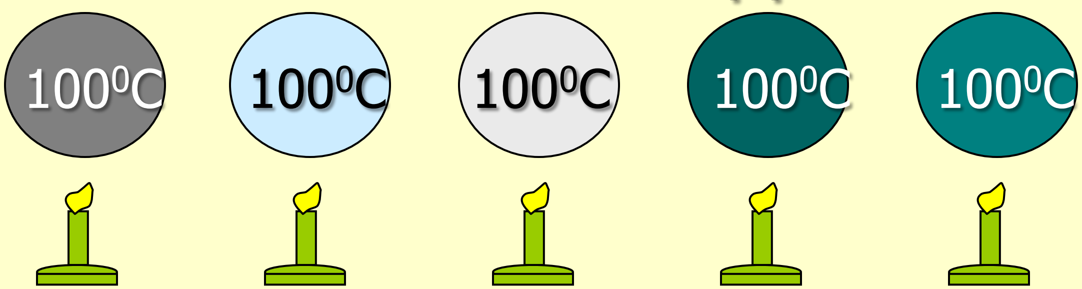
The larger mass experiences a smaller increase in temperature.



Heat Capacity

The <u>heat capacity</u> of a substance is the heat required to raise the temperature a unit degree.

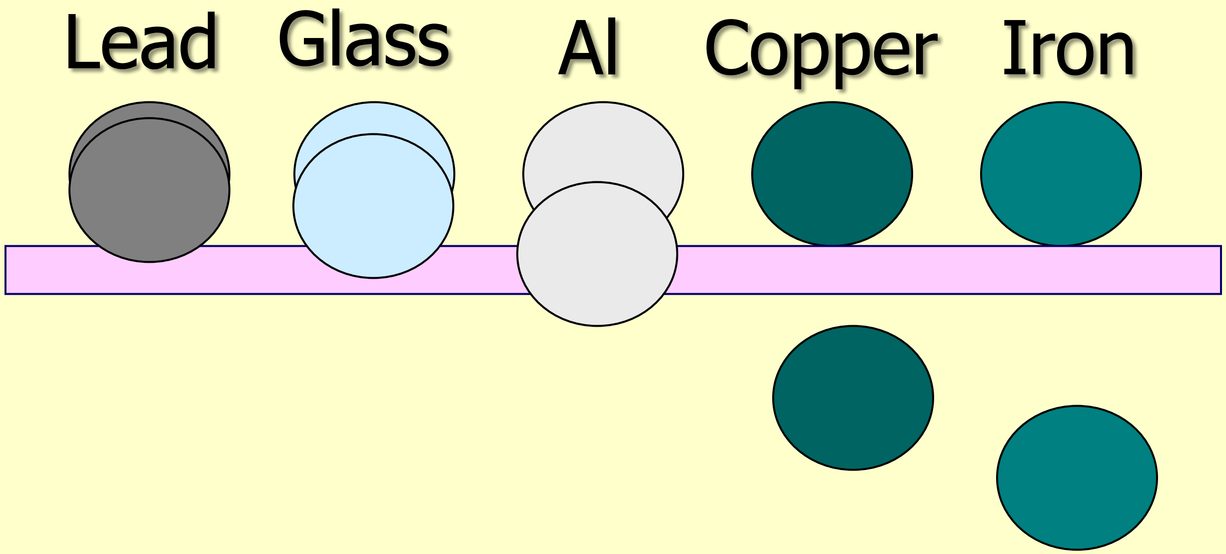
Lead	Glass	Al	Copper	Iron
37 s	52 s	60 s	83 s	90 s



Heat capacities based on time to heat from zero to 100^{0}C. Which has the greatest heat capacity?

Heat Capacity (Continued)

All at 100^0C placed on Paraffin Slab



Iron and copper balls melt all the way through; others have lesser heat capacities.

Specific Heat Capacity

The specific heat capacity of a material is the quantity of heat needed to raise the temperature of a unit mass through a unit degree.

$$c = \frac{Q}{m\Delta t}; \qquad Q = mc\Delta t$$

Water: c = 1.0 cal/g C^0 or 1 Btu/lb F^0 or 4186 J/kg K

Copper: c = 0.094 cal/g C^0 or 390 J/kg K

<u>Comparison of Heat Units</u> : How much heat is needed to raise 1-kg of water from 0^0 to 100^0C?

The mass of one kg of water is: 1 kg = 1000 g = 0.454 lb_m

$$Q = mc\Delta t$$

1 lb_m = 454 g

For water: c = 1.0 cal/g C^0 or 1 Btu/lb F^0 or 4186 J/kg K

The heat required to do this job is:

10,000 cal   10 kcal

39.7 Btu   41, 860 J

Problem Solving Procedure

1. Read problem carefully and draw a rough sketch.
2. Make a list of all given quantities
3. Determine what is to be found.
4. Recall applicable law or formula and constants.

$$c = \frac{Q}{m\Delta t}; \qquad Q = mc\Delta t$$

Water: c = 1.0 cal/g C^0 or 1 Btu/lb F^0 or 4186 J/kg K

5. Determine what was to be found.

Example 1: A **500-g** copper coffee mug is filled with **200-g** of coffee. How much heat was required to heat cup and coffee from **20** to **96^{0}C**?

1. Draw sketch of problem.

2. List given information.

Mug mass $m_m = 0.500$ kg

Coffee mass $m_c = 0.200$ kg

Initial temperature of coffee and mug: $t_0 = 20^0$C

Final temperature of coffee and mug: $t_f = 96^0$C

3. List what is to be found: Total heat to raise temperature of coffee (water) and mug to 96^{0}C.

Example 1(Cont.): **How much heat needed to heat cup and coffee from 20 to 96^0C?**

$m_m = 0.2$ kg; $m_w = 0.5$ kg.

4. Recall applicable formula or law:

Heat Gain or Loss: $Q = mc\,\Delta t$

5. Decide that TOTAL heat is that required to raise temperature of mug and water (coffee). Write equation.

$$Q_T = m_m c_m \,\Delta t + m_w c_w \,\Delta t$$

6. Look up specific heats in tables:

Copper: $c_m = 390$ J/kg C^0

Coffee (water): $c_w = 4186$ J/kg C^0

Example 1(Cont.): **How much heat needed to heat cup and coffee from 20 to 96^{0}C?** $m_c = 0.2$ **kg;** $m_w = 0.5$ **kg.**

7. Substitute info and solve problem:

Copper: $c_m = 390$ J/kg C^0

Coffee (water): $c_w = 4186$ J/kg C^0

$$Q_T = m_m c_m \Delta t + m_w c_w \Delta t$$

Water: (0.20 kg)(4186 J/kgC^0)(76 C^0)

$\Delta t = 96^0C - 20^0C = 76\ C^0$

Cup: (0.50 kg)(390 J/kgC^0)(76 C^0)

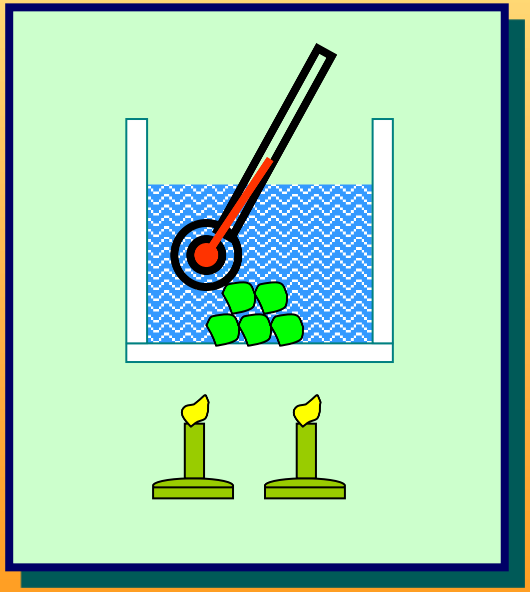
$Q_T = 63{,}600$ J + 14,800 J

$Q_T = 78.4$ kJ

A Word About Units

The substituted units must be consistent with those of the chosen value of specific heat capacity.

For example: Water c_w = 4186 J/kg C^0 or 1 cal/g C^0



$$Q = m_w c_w \Delta t$$

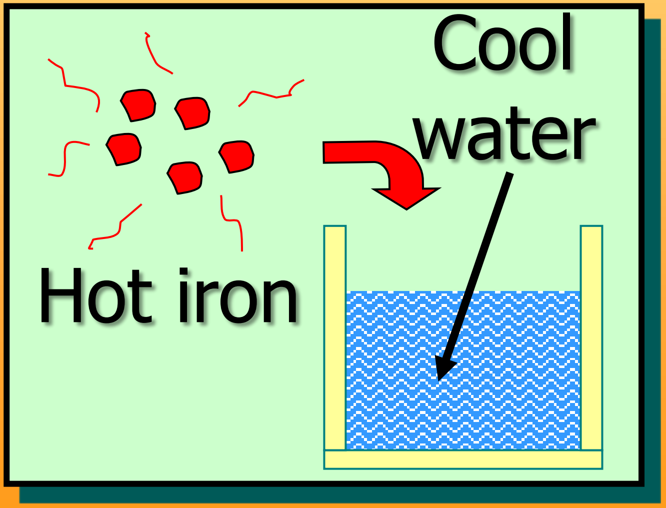
If you use 1 cal/g C^0 for *c*, then *Q* must be in calories, and *m* must be in grams.

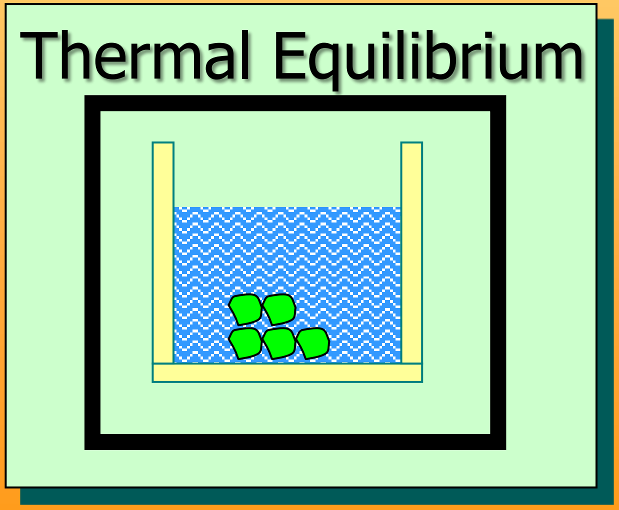
of the constant *c*.

Conservation of Energy

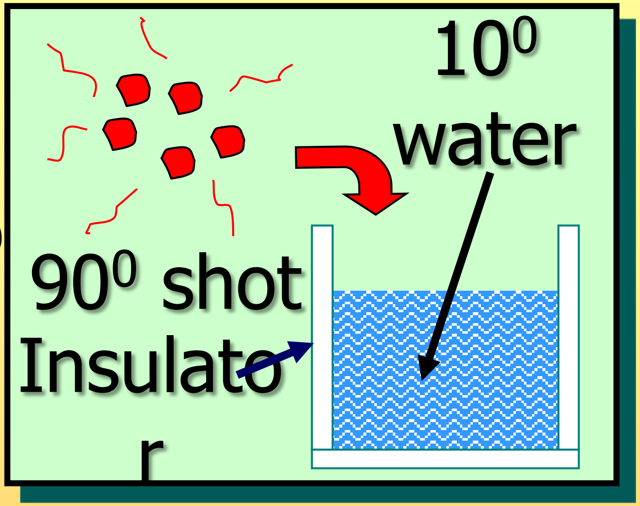
Whenever there is a transfer of heat within a system, the heat lost by the warmer bodies must equal the heat gained by the cooler bodies:

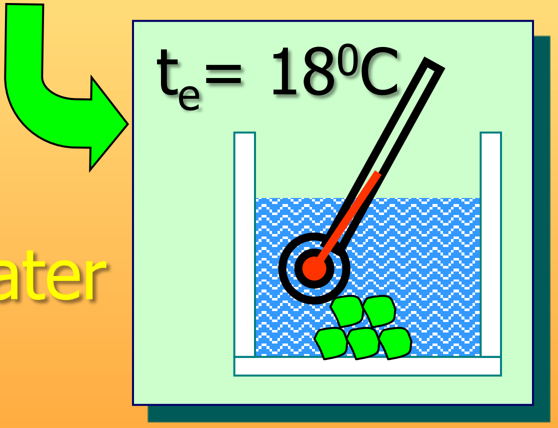
$$\Sigma \text{ (Heat Losses)} = \Sigma \text{ (Heat Gained)}$$





Example 2: **A handful of copper shot is heated to 90^{0}C and then dropped into 80 g of water in an insulated cup at 10^{0}C. If the equilibrium temperature is 18^{0}C, what was the mass of the copper?**





c_w = 4186 J/kg C^0; c_s = 390 J/kg C^0

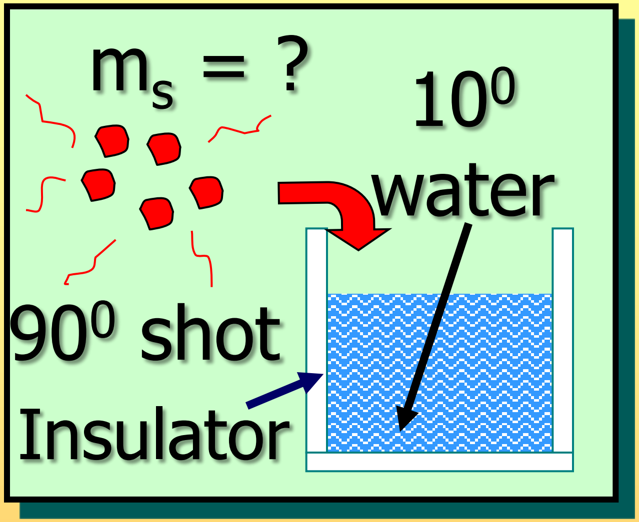
m_w = 80 g; t_w= 10^{0}C; t_s = 90^{0}C

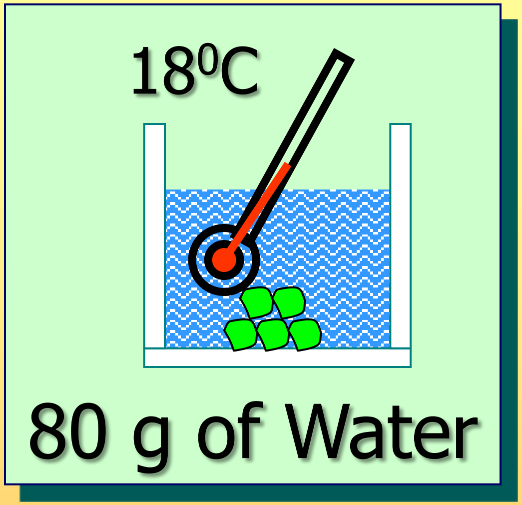
Heat lost by shot = heat gained by water

$$m_s c_s(90^0\text{C} - 18^0\text{C}) = m_w c_w(18^0\text{C} - 10^0\text{C})$$

Note: Temperature differences are [High - Low] to insure absolute values (+) lost and gained.

Example 2: (Cont.)





Heat lost by shot = heat gained by water

$$m_s c_s(90^0\text{C} - 18^0\text{C}) = m_w c_w(18^0\text{C} - 10^0\text{C})$$

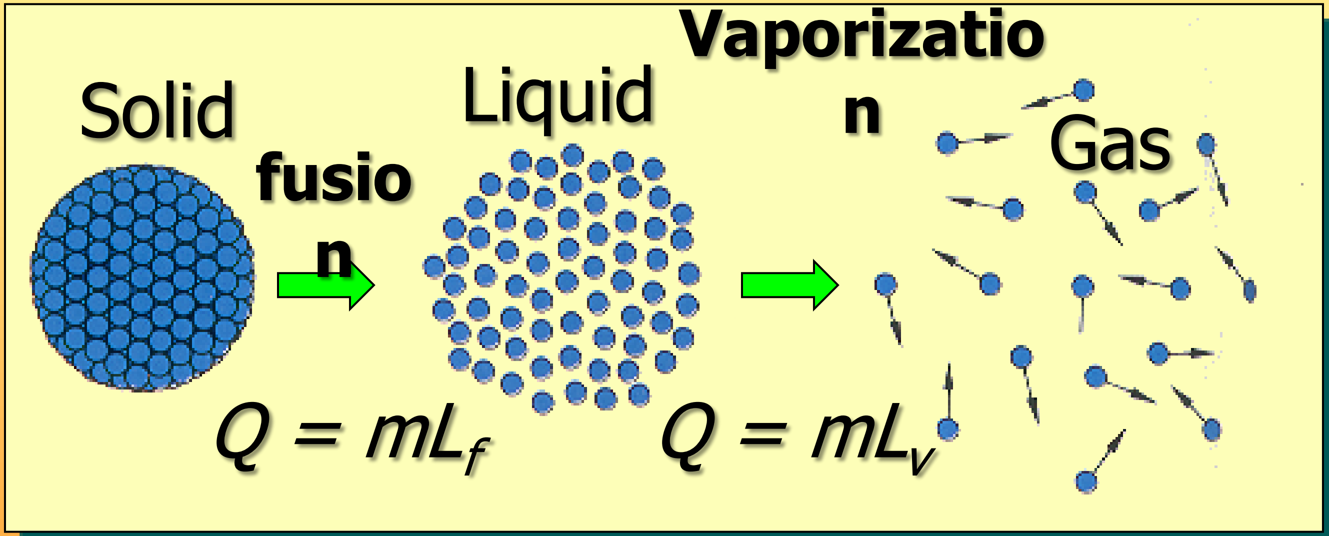
$$m_s(390\text{ J/kgC}^0)(72\text{ C}^0) = (0.080\text{ kg})(4186\text{ J/kgC}^0)(8\text{ C}^0)$$

$$m_s = \frac{2679\text{ J}}{28{,}080\text{ J/kg}} = 0.0954\text{ kg}$$

$$m_s = 95.4\text{ g}$$

Change of Phase

When a change of phase occurs, there is only a change in potential energy of the molecules. The temperature is constant during the change.



Terms: Fusion, vaporization, condensation, latent heats, evaporation, freezing point, melting point.

Change of Phase

The latent heat of fusion (L_f) of a substance is the heat per unit mass required to change the substance from the solid to the liquid phase of its melting temperature.

$$L_f = \frac{Q}{m}$$

For Water: L_f = 80 cal/g = 333,000 J/kg

The latent heat of vaporization (L_v) of a substance is the heat per unit mass required to change the substance from a liquid to a vapor at its boiling temperature.

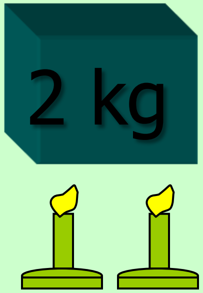
$$L_v = \frac{Q}{m}$$

For Water: L_v = 540 cal/g = 2,256,000 J/kg

Melting a Cube of Copper

The heat Q required to melt a substance at its melting temperature can be found if the mass and latent heat of fusion are known.

$$Q = mL_v$$



What Q to melt copper?

L_f = 134 kJ/kg

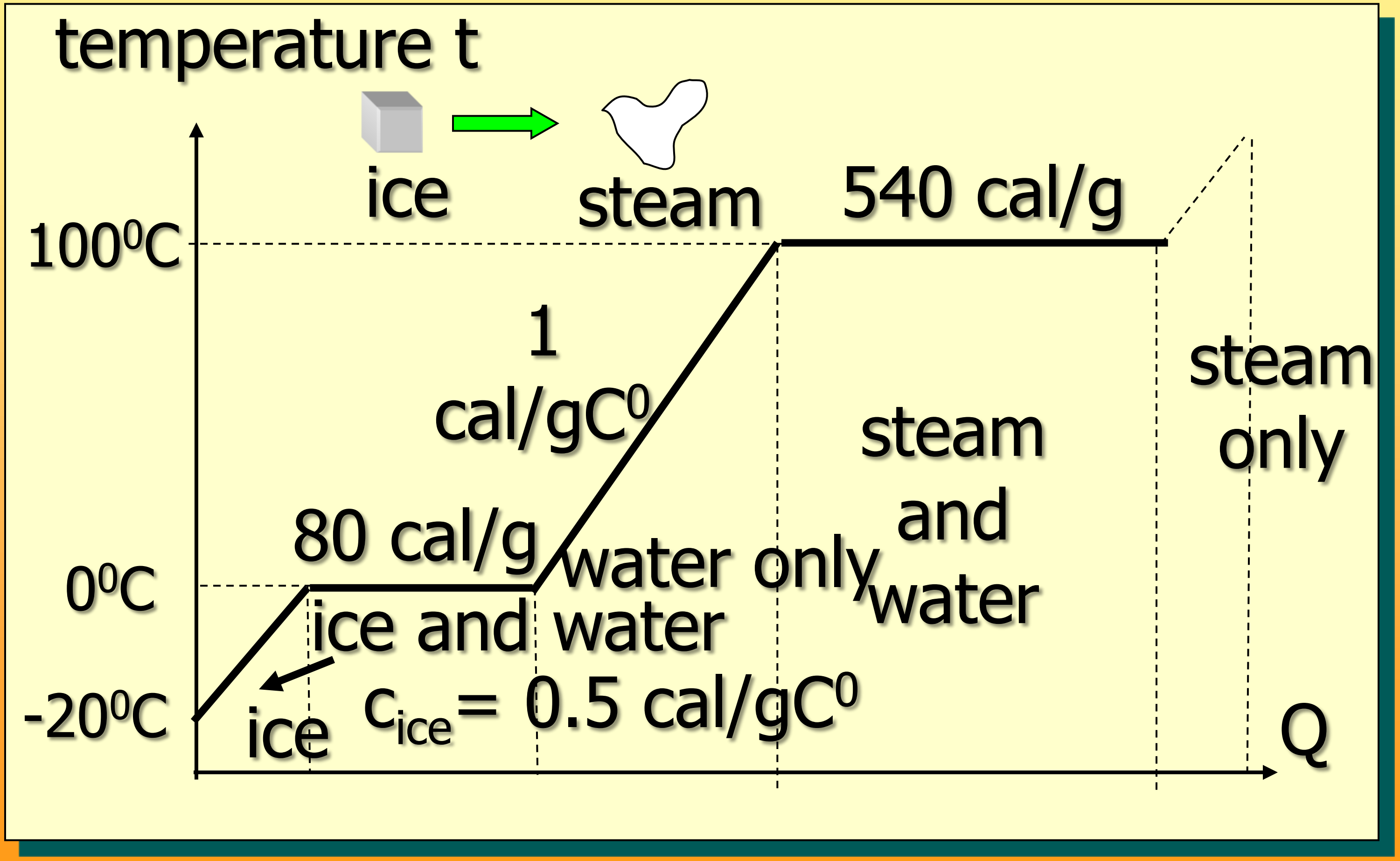
Example : To completely melt 2 kg of copper at 1040^{0}C, we need:

$$Q = mL_f = (2\ \text{kg})(134{,}000\ \text{J/kg})$$

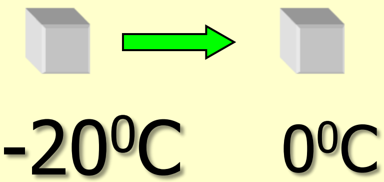
$$Q = 268\ \text{kJ}$$

Example 3: **How much heat is needed to convert 10 g of ice at -20^{0}C to steam at 100^{0}C?**

First, let's review the process graphically as shown:



Example 3 (Cont.): **Step one is Q_1 to convert 10 g of ice at -20^0C to ice at 0^0C (no water yet).**



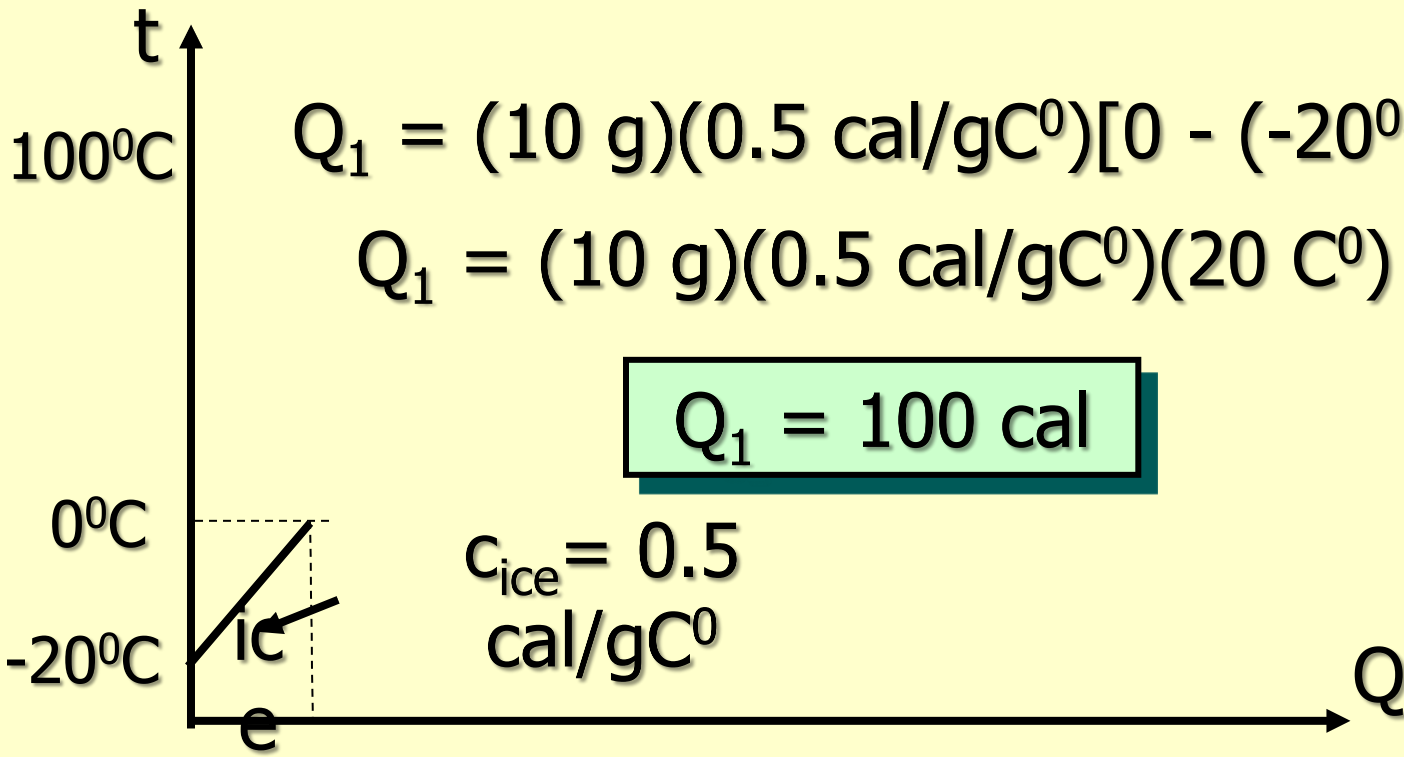
Q_1 to raise ice to 0^0C: $Q_1 = mc\Delta t$

$$Q_1 = (10\text{ g})(0.5\text{ cal/gC}^0)[0 - (-20^0C)]$$

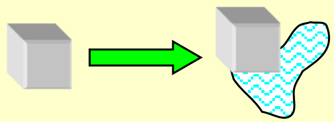
$$Q_1 = (10\text{ g})(0.5\text{ cal/gC}^0)(20\text{ C}^0)$$

$$\boxed{Q_1 = 100\text{ cal}}$$

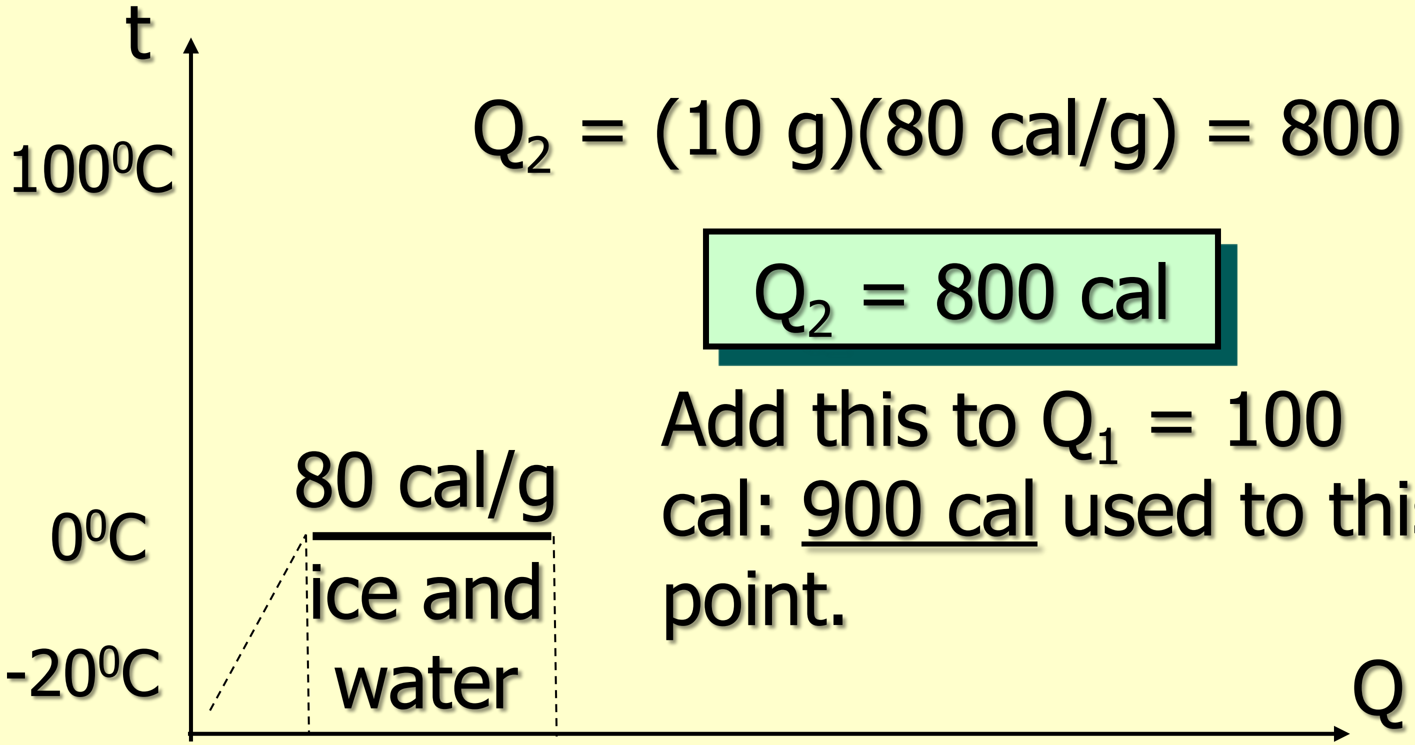
$c_{ice} = 0.5$ cal/gC^0



Example 3 (Cont.): Step two is Q_2 to convert 10 g of ice at 0^0C to water at 0^0C.



Q_2 to melt 10 g of ice at 0^0C: $Q_2 = mL_f$

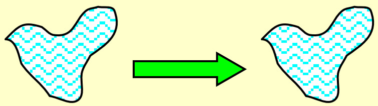


$$Q_2 = (10\text{ g})(80\text{ cal/g}) = 800\text{ cal}$$

$$\boxed{Q_2 = 800\text{ cal}}$$

Add this to $Q_1 = 100$ cal: 900 cal used to this point.

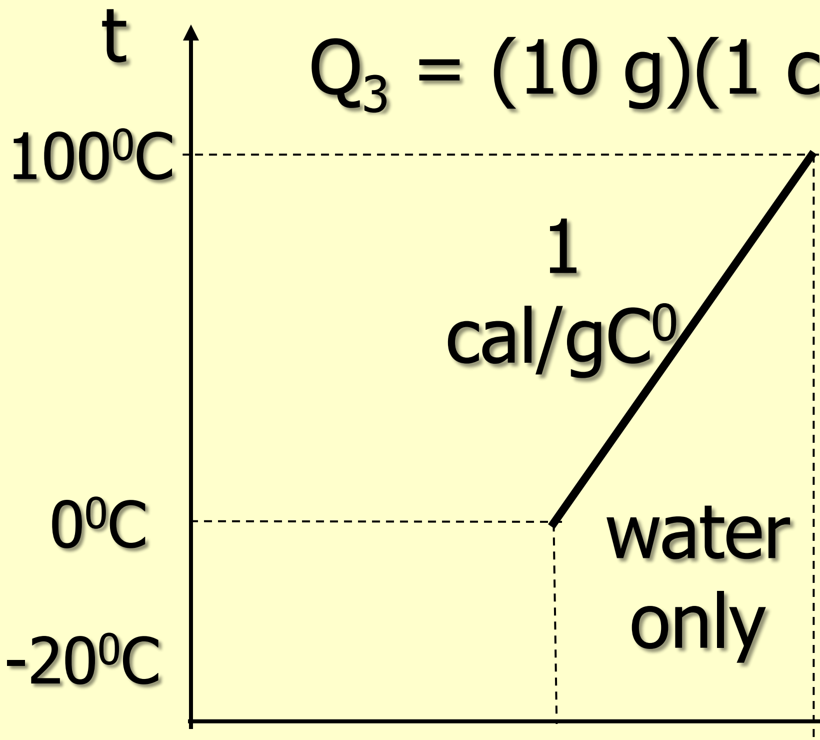
Example 3 (Cont.): Step three is Q_3 to change 10 g of water at 0^0C to water at 100^0C



Q_3 to raise water at 0^0C to 100^0C.

$Q_3 = mc\Delta t$; $c_w = 1$ cal/gC^0

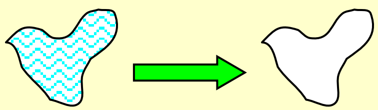
$$Q_3 = (10\text{ g})(1\text{ cal/gC}^0)(100^0C - 0^0C)$$



$$\boxed{Q_3 = 1000\text{ cal}}$$

Total = $Q_1 + Q_2 + Q_3$
= 100 +900 + 1000 = 1900 cal

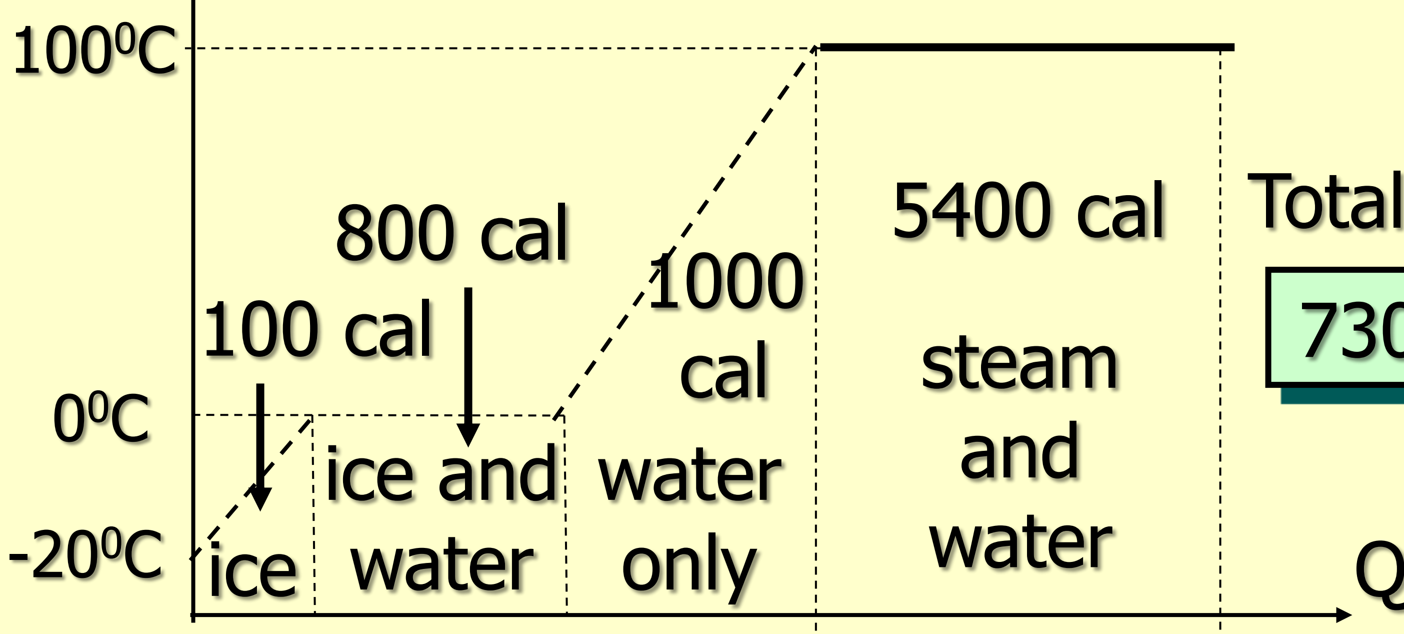
Example 3 (Cont.): Step four is Q_4 to convert 10 g of water to steam at 100^0C? ($Q_4 = mL_v$)



vaporization

Q_4 to convert all water at 100^0C to steam at 100^0C. ($Q = mL_v$)

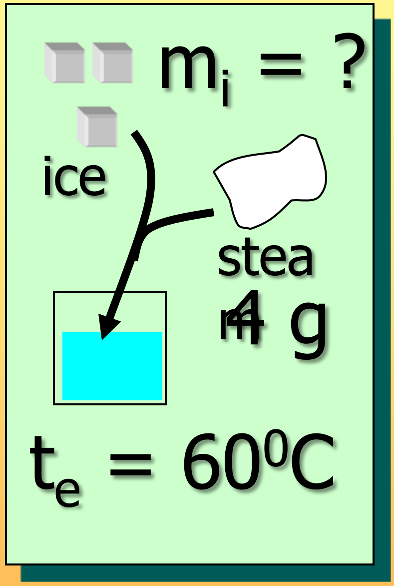
$$Q_4 = (10\text{ g})(540\text{ cal/g}) = 5400\text{ cal}$$



Total Heat:

7300 cal

Example 4: **How many grams of ice at 0⁰C must be mixed with four grams of steam in order to produce water at 60⁰C?**



Ice must melt and then rise to 60^0C.
Steam must condense and drop to 60^0C.

Total Heat Gained = Total Heat Lost

$$m_iL_f + m_ic_w\Delta t = m_sL_v + m_sc_w\Delta t$$

Note: All losses and gains are absolute values (positive).

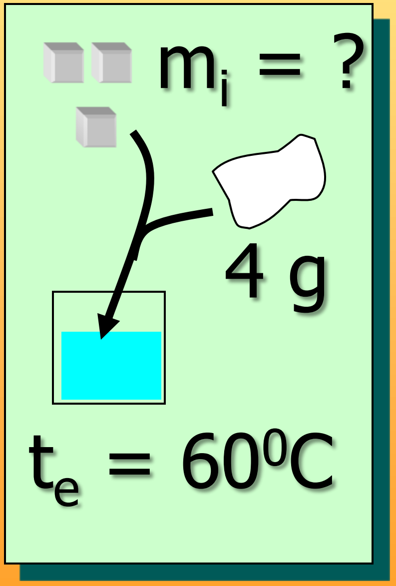
Total Gained: $m_i(80 \text{ cal/g}) + m_i(1 \text{ cal/gC}^0)(60 \text{ C}^0 - 0^0\text{C})$

Lost: $(4 \text{ g})(540 \text{ cal/g}) + (4 \text{ g})(1 \text{ cal/gC}^0)(100 \text{ C}^0 - 60^0\text{C})$

Example 4 (Continued)

Total Gained: $m_i(80\ \text{cal/g}) + m_i(1\ \text{cal/gC}^0)(60\ \text{C}^0)$

Total Lost: $(4\ \text{g})(540\ \text{cal/g}) + (4\ \text{g})(1\ \text{cal/gC}^0)(40\ \text{C}^0)$



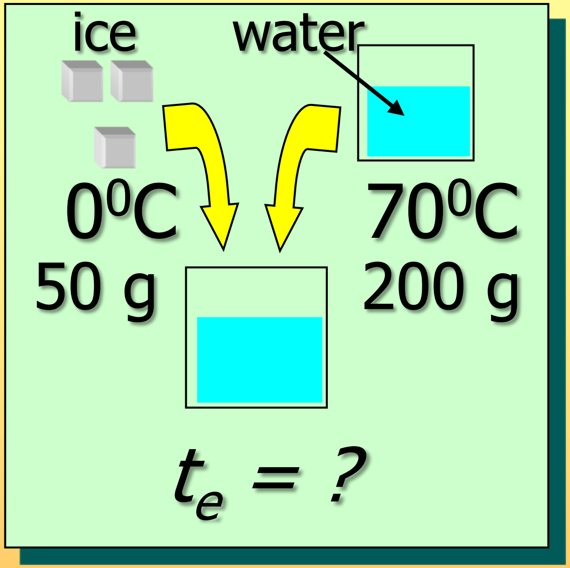
Total Heat Gained = Total Heat Lost

$$80m_i + 60m_i = 2160\ \text{g} + 160\ \text{g}$$

$$m_i = \frac{2320\ \text{g}}{140}$$

$$m_i = 16.6\ \text{g}$$

Example 5: **Fifty grams of ice are mixed with 200 g of water initially at 70^0C. Find the equilibrium temperature of the mixture.**



Ice melts and rises to t_e

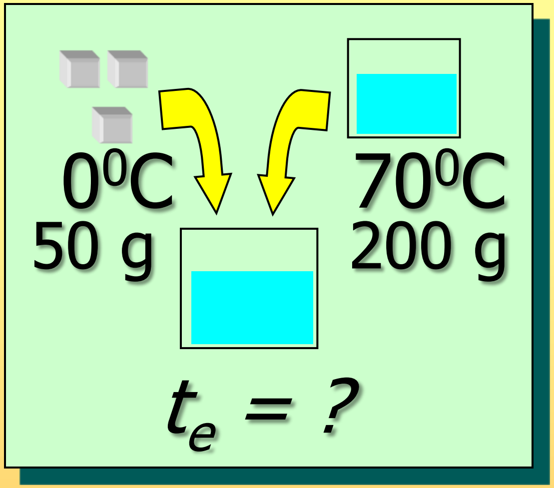
Water drops from 70 to t_e.

Heat Gained: $m_i L_f + m_i c_w \Delta t$; $\Delta t = t_e - 0^0\text{C}$

Gain = (50 g)(80 cal/g) + (50 g)(1 cal/gC^0)(t_e - 0^0C)

Gain = 4000 cal + (50 cal/g)t_e

Example 5 (Cont.):



Gain = 4000 cal + (50 cal/g)t_e

Heat Lost $= m_w c_w \Delta t$

$\Delta t = 70^0\text{C} - t_e$ [high - low]

Lost = (200 g)(1 cal/gC^0)(70^0C- t_e)

Lost = 14,000 cal - (200 cal/C^0) t_e

Heat Gained Must Equal the Heat Lost:

$$4000 \text{ cal} + (50 \text{ cal/g})t_e = 14{,}000 \text{ cal} - (200 \text{ cal/C}^0)\, t_e$$

Example 5 (Cont.):

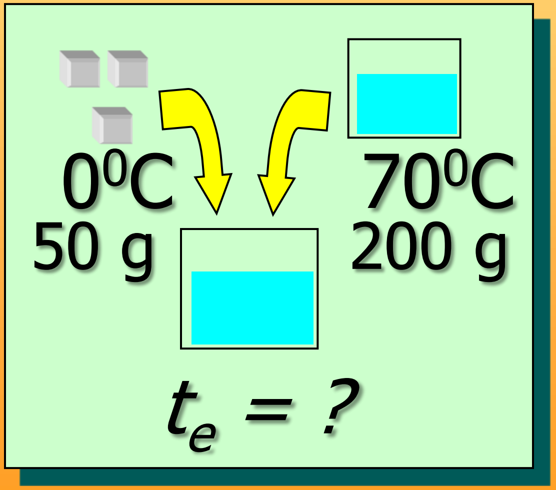
Heat Gained Must Equal the Heat Lost:

$4000 \text{ cal} + (50 \text{ cal/g})t_e = 14{,}000 \text{ cal} - (200 \text{ cal/C}^0)\ t_e$

Simplifying, we have: $(250 \text{ cal/C}^0)\ t_e = 10{,}000 \text{ cal}$

$$t_e = \frac{10{,}000 \text{ cal}}{250 \text{ cal/C}^0} = 40^0\text{C}$$

$t_e = 40^0\text{C}$



Summary of Heat Units

One <u>calorie</u> (1 cal) is the quantity of heat required to raise the temperature of 1 g of water by 1 C^0.

One <u>kilocalorie</u> (1 kcal) is the quantity of heat required to raise the temperature of 1 kg of water by 1 C^0.

One <u>British thermal unit</u> (Btu) is the quantity of heat required to raise the temperature of 1 lb of water by 1 F^0.

Summary: Change of Phase

The latent heat of fusion (L_f) of a substance is the heat per unit mass required to change the substance from the solid to the liquid phase of its melting temperature.

$$\boxed{L_f = \frac{Q}{m}}$$

For Water: L_f = 80 cal/g = 333,000 J/kg

The latent heat of vaporization (L_v) of a substance is the heat per unit mass required to change the substance from a liquid to a vapor at its boiling temperature.

$$\boxed{L_v = \frac{Q}{m}}$$

For Water: L_v = 540 cal/g = 2,256,000 J/kg

Summary: Specific Heat Capacity

The specific heat capacity of a material is the quantity of heat to raise the temperature of a unit mass through a unit degree.

$$c = \frac{Q}{m\Delta t}; \qquad Q = mc\Delta t$$

Summary: Conservation of Energy

Whenever there is a transfer of heat within a system, the heat lost by the warmer bodies must equal the heat gained by the cooler bodies:

$$\Sigma\ (\text{Heat Losses}) = \Sigma\ (\text{Heat Gained})$$

Summary of Formulas:

$$c = \frac{Q}{m\Delta t}; \qquad Q = mc\Delta t$$

Σ (Heat Losses) = Σ (Heat Gained)

$$L_f = \frac{Q}{m}; \qquad Q = mL_f$$

$$L_v = \frac{Q}{m}; \qquad Q = mL_v$$

Thanks…