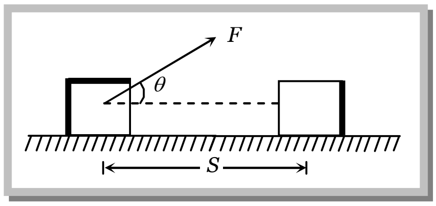
Work, Power and Energy

Work

Work done by a constant force is defined as the product of force (F) and the actual distance (s) moved by the body in the direction of application of force or work done by a constant force is given by the dot product of force and displacement.



Formula : $W = \vec{F} \cdot \vec{S}$, $W = F\,S\cos\theta$

Obviously, work is a scalar quantity *i.e.* it has magnitude only but no direction. However work done by a force can be positive, negative or zero.

Dimension : $\left[ML^2T^{-2}\right]$

Unit : (1) **Absolute units :** *Joule* (in S.I. system), *Erg* (in C.G.S. system)

1 *Joule* = 10^7 *Erg*

(2) **Gravitational units :** *Kilogram-meter* (in S.I. system), *gm-centimetre* (in C.G.S. system)

1 *Kilogram-meter* = 9.8 *Joule*

1 *gram-centimetre* = 980 *Erg*

Positive Negative and Zero Work

(1) **Positive work :** when θ is acute ($< 90^\circ$), $\cos\theta$ is positive hence work done is positive

Example : (i) When a body falls freely under the action of gravity. $\theta = 0^\circ$, $\cos\theta = \cos 0^\circ = +1$. Therefore work done by Gravity on a body falling freely is positive.

(ii) When a lawn roller is pulled by applying a force along the handle at an acute angle. Work done by the applied force is positive.

(iii) When a spring is stretched, work done by the stretching force is positive.

(2) **Negative work :** when θ is obtuse ($> 90^\circ$), $\cos\theta$ is negative. Hence work done is negative.

Example : (i) When a body is made to slide over a rough surface, the work done by the frictional force is negative.

(ii) When a positive charge is moved towards another positive charge. The work done by electrostatic force between them is negative.

(3) **Zero work :** When force applied $\vec{F}$ or the displacement $\vec{S}$ or both are zero, work done $W = FS\cos\theta$ is zero.

Again when angle θ between $\vec{F}$ and $\vec{S}$ is 90°, $\cos\theta = \cos 90^\circ = 0$ $\therefore$ Work done is zero.

Example : (i) When a coolie travels on a platform with a load on his head, work done by the coolie is zero.

(ii) When a body moves along the circular path with the help of a string, the work done by the tension in the string is zero.

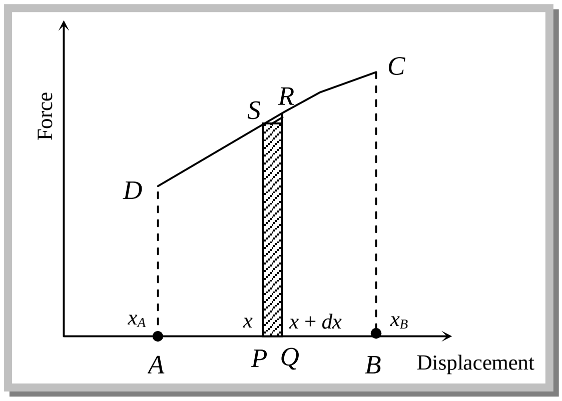
(iii) When a person does not move from his position but he may be holding any amount of heavy load. The work done is zero.

Work Done by a Variable Force

There are many situations in which a body may be moving under the effect of a varying force.

Example: (1) When a body is moved away from the centre of earth the magnitude of the gravitational force continuously decreases.

(2) When a body attached at the end of a spring and perform simple harmonic motion, restoring force in the spring is a variable force.



∴ Small amount of work done in moving the body from P to Q is

$dw = Fdx = (PS)\,(PQ)$ = Area of strip

Total work done $W = \int_{xA}^{xB} Fdx \Rightarrow W = \int_{xA}^{xB}$ Area of strip ($PQRS$)

W = Total area under the curve between F and x axis Hence work done by a variable force is numerically equal to the area under the force curve and displacement axis.

Work Done in Stretching a Spring

When the spring is compressed or elongated, it tends to recover its original length, on account of elasticity. The force is trying to bring the spring back to original configuration is called restoring force.

Within the elastic limit, the restoring force (F) is directly proportional to the displacement (x)

$F \propto -x = -Kx$ where K is a constant of the spring and is called spring constant.

If $x = 1, F = -K$ or $K = F$

Hence "spring constant is numerically equal to force required to produce unit displacement (compression/ extension) in the spring". Its value depends upon the nature of the spring. The negative sign indicates that the restoring force directed always towards the equilibrium position.

Work done in stretching a spring = potential energy of the spring = $W = \frac{1}{2}kx^2 = \frac{F^2}{2K} = \frac{1}{2}Fx$

Conservative and Non-conservative Forces

(1) **Conservative forces :** A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final position of the body, and not on the nature of path followed between the initial and final positions and for a round trip this work done is always zero.

Example : (i) Gravitational force (ii) Electrostatic force (iii) Magnetic force

(2) **Non-conservative forces :** A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions. And for complete cycle this work done can never be a zero.

Example : (i) Frictional force (ii) Viscous force

Power

Power of a body is defined as the rate at which the body can do the work.

Formula : Power = Rate of doing work $= \frac{\text{work}}{\text{time}} = \frac{W}{t} = \frac{\vec{F} \cdot \vec{s}}{t} = \vec{F} \cdot \vec{v}$

Dimension : $[ML^2T^{-3}]$

Units : *Watt* or *Joule/sec* (In M.K.S system), 1 *kilo watt* = 10^3 *watt*, 1 *mega watt* = 10^6 *watt*

Erg/sec (in C.G.S. system)

1 *watt* = 1 *J/sec* = 10^7 *erg/sec*, 1 H.P. = 746 *watt* = 746 *Joule/sec*, 1 K.W. = 10^3 *watt* = 1.34 HP

Energy

Energy of a body is defined as the capacity or ability of the body to do the work. Since energy of a body is the total quantity of work done. It is a scalar quantity.

Dimension : $[ML^2T^{-2}]$

Unit : *Joule* (in S.I. system) and erg (in C.G.S. system)

Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Example : (i) Kinetic energy of air is used to run windmills.

(ii) Kinetic energy of running water is used to run water mills.

(iii) Kinetic energy of hammer is made use of in driving a nail into a piece of wood.

(iv) A bullet fired from a gun can pierce a target due to its kinetic energy.

(v) If a body of mass m is moving with velocity v then kinetic energy possessed by the body $KE = \frac{1}{2}mv^2$.

Relation between kinetic energy and linear momentum : If E = Kinetic energy, v = velocity, P = Linear momentum, m = mass of the body then, $P = mv = \sqrt{2mE}$ and $E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{1}{2}Pv$,

Work-Energy Principle

According to this principle , work done by in displacing a body measures the change in kinetic energy of the body. If the moves with velocity u initially and by applying a constant force it velocity finally becomes v.

Then workdone on the body W = increment in kinetic energy $\Rightarrow$ $W = \frac{1}{2}m(v^2 - u^2)$

When kinetic energy of a body decreases, work done on the body must be negative and vice-versa.

Conversely, when work is positive, *KE* will increase and when work is negative *KE* will decrease.

Principle of Conservation of Energy

It states that the energy can neither be created nor be destroyed but can only be converted from one form to another. In other words-"The total energy of an isolated system always remains constant".

Mass Energy Equivalence

In the year 1905, Einstein made an incredible discovery that energy can be transformed into mass and vice-versa *i.e.* mass can be transformed into energy.

$E = mc^2$ where m = mass that disappears; E = energy that appears; c = velocity of light in vacuum.

Potential Energy

It is the energy possessed by a body by virtue of its position or configuration (shape or size). Potential energy are of different types :

(1) **Gravitational potential energy :** Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

(i) Gravitational potential energy = mgh [if $h << R$]

(ii) Gravitation potential energy = $\dfrac{mgh}{1+\frac{h}{r}}$

here m = mass of the body; g = acceleration due to gravity; h = height, If h is comparable to R

(iii) If two bodies of mass m_1 and m_2 are situated r distance a part then the Gravitational Potential Energy

$$U = \frac{-Gm_1m_2}{r}$$

(2) **Elastic potential energy** (Potential Energy of a spring) **:** It is the energy associated with the state of compression or expansion of an elastic spring , and is also called elastic potential energy.

If k is the spring constant of spring and force F is applied on it so that its length changes by x then some work has to be done. And this work is stored in the form of potential energy of spring.

$$U = \frac{1}{2}kx^2 \Rightarrow U = \frac{1}{2}Fx \Rightarrow U = \frac{F^2}{2k}$$

Energy graph for spring : For a spring when a horizontal spring performs SHM. It possess kinetic and potential energy in different ratio at different positions.

$A \rightarrow$ Extreme left (Amplitude) position.

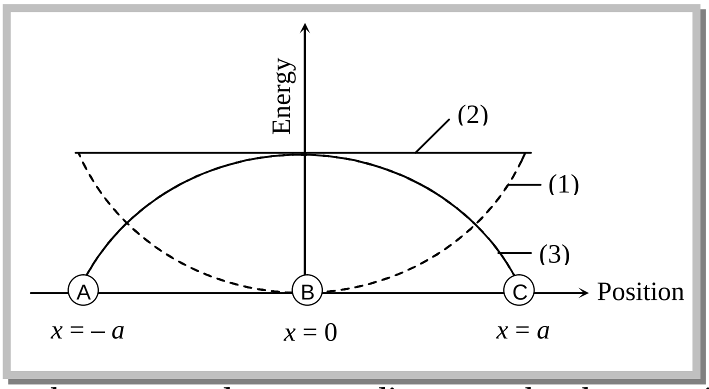
$B \rightarrow$ Mean position

$C \rightarrow$ Extreme right (Amplitude) position.

Graph No. 1 $\rightarrow$ Potential Energy Graph.

Graph No. 2 $\rightarrow$ Total Energy Graph.

Graph No. 3 $\rightarrow$ Kinetic Energy Graph.



(3) **Electrostatic potential energy :** When two charges q_1 and q_2 are at distance r the electrostatic potential energy of system

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r};$$

U = positive when charges are of similar nature

U = Negative when charge are of opposite nature.

Law of Conservation of Liner Momentum

In an isolated system, the vector sum of the linear momenta of all the bodies of the system is conserved and is not affected due to their mutual action and reaction.

Let an isolated system consisting of n bodies of masses m_1, m_2,-------m_n moving with velocities $\vec{v}_1, \vec{v}_2 - - - \vec{v}_n$ respectively.

Total linear momenta of the system $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + - - - - - - = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + - - - - -$

by the Newton's second law $F = \frac{dp}{dt}$ and for isolated system $F = 0$ so $\vec{p}$ = constant

$\therefore m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...=$ constant

Collision

Collision between two particles is said to occur when they actually strike against each other or the path of motion of one particle or body is affected by the other.

Collision are for types : Elastic collision, Inelastic collision

(1) **Elastic collision :** A collision in which there is absolutely no loss of kinetic energy is called an elastic collision.

Example : (i) Collision between atomic and sub- atomic particles is elastic collisions.

(ii) Collision between two ivory balls can be taken as an elastic collision.

Characteristics : Linear momentum, Total energy, Kinetic energy is conserved and Force involved during elastic collision must be conservation force.

(2) **Inelastic collision :** A collision in which there occurs some loss of kinetic energy is called an inelastic collision.

Example : If two bodies stick to each other, after colliding , the collision is said to be perfectly in elastic as in case of mud thrown on a wall sticks to the wall.

Characteristics : Linear momentum and Total energy of the system is conserved but Kinetic energy is not conserved. A part of mechanical energy (i.e. K.E.) is converted into some form of energy like heat and sound energy.

Coefficient of Restitution

In actual practice, collisions between all real object are neither perfectly elastic nor they are perfectly inelastic. They are called imperfect or semi elastic collisions.

The degree of elasticity of a collision is determined by a quantity called coefficient of restitution or coefficient of resilience of the collision.

It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision.

$$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of apporach (before collision)}}$$

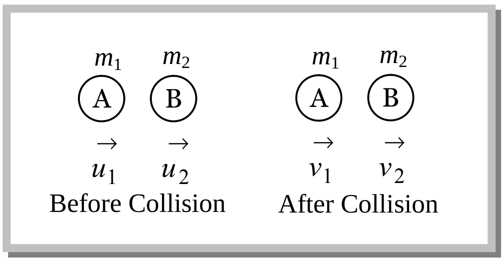
For perfectly elastic collision $e = 1$, for perfectly inelastic collision $e = 0$, for all other collision $0 < e < 1$

Elastic Collision in One Dimension

Let two balls A and B of masses m_1 and m_2 are moving initially along the same straight line with velocities $u_1$and u_2 respectively.

when $u_1 > u_2$ relative velocity of approach before collision = $u_1 - u_2$

After perfectly elastic collision v_1 is the velocity of A and v_2 is the velocity of B along the same straight line.



When $v_2 > v_1$ the bodies separate after collision relative velocity of separation after collision $= v_2 - v_1$

By the law of conservation of linear momentum $m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$

By the law of conservation of kinetic energy $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + m_2 u_2^2$

After solving these two equations we get $v_2 - v_1 = u_1 - u_2$

or $\frac{v_2 - v_1}{u_1 - u_2} = 1 \Rightarrow \frac{\text{Relative velocity of recession}}{\text{Relative velocity of approach}} = 1 \Rightarrow e = 1$

After solving above equations we can get : $v_1 = \frac{(m_1 - m_2)}{m_1 + m_2} u_1 + \frac{2 m_2 u_2}{m_1 + m_2}$

and $v_2 = \frac{(m_2 - m_1)}{m_1 + m_2} u_2 + \frac{2 m_1 u_1}{m_1 + m_2}$

Special condition : (1) When masses of two bodies are equal *i.e.* $m_1 = m_2$

$v_1 = u_2; v_2 = u_1$

It means when two bodies of equal masses undergo elastic collision in one dimension, there velocities are just interchanged.

(2) When the target body B is initially at rest *i.e.* $u_2 = 0$

$v_1 = \frac{(m_1 - m_2) u_1}{m_1 + m_2}$ and $v_2 = \frac{2 m_1 u_1}{m_1 + m_2}$

(i) When masses of two bodies are equal *i.e.,* $m_1 = m_2$

$v_1 = 0; \quad v_2 = u_1$

It means body a comes to rest and body B starts moving with the initial velocity of A

(ii) When the body B at rest is very heavy *i.e.,* $m_2 >> m_2$

$v_1 = -u_1; \quad v_2 = 0$

It means when a light body A collides against a heavy body B at rest, A rebounds with its own velocity and B continues to be at rest.

Example : When a ball strikes the floor elastically, it rebounds up to a same height from which it was thrown.

(iii) When body B at rest has negligible mass *i.e.,* $m_2 << m_1$

$v_1 = u_1; \quad v_2 = 2u_1$

It means when a heavy body A under goes an elastic collision with a light body B at rest, the body A keeps on moving with the same velocity of its own and the body B starts moving with double the initial velocity of A.

States of Equilibrium

Stable	Unstable	Neutral
When displaced from equilibrium position, particle tends to come back.	When displaced from equilibrium position, particle tends to move away from equilibrium position.	Particle always remains in the state of equilibrium irrespective of any displacement.
Potential energy U = minimum	Potential energy U = maximum	Potential energy U = constant

$\frac{d^2U}{dx^2}$ = positive	$\frac{d^2U}{dx^2}$ = negative	$\frac{d^2U}{dx^2} = 0$
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$

Some Important Facts and Formulae

(1) If K_i and K_f be the initial and final energies of the ball of mass M_1, then fractional decrease in energy is given by

$$\frac{K_i - K_f}{K_i} = 1 - \frac{v_1^2}{u_1^2}$$

If $m_2 = nm_1$ and $u_2 = 0$ then $\frac{K_i - K_f}{K_i} = \frac{4n}{(1+n)^2}$

(2) The loss of kinetic energy during inelastic collision is given by

$$\Delta E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(e^2 - 1)(u_1 - u_2)^2$$

(3) If a body is dropped from a height h_0 and it strikes the ground with velocity v_0. After the inelastic collision it rise to a height h_1. If v_1 is the velocity with which the body rebounds

$$e = \frac{v_1}{v_0} = \left[\frac{2gh_1}{2gh_0}\right]^{1/2} = \left[\frac{h_1}{h_0}\right]^{1/2} \Rightarrow h_1 = h_0 e^2$$

Note : ❑ Distance covered by body in n^{th} collision $h_n = h_0 e^{2n}$

❑ Total time taken by body in coming to rest $= \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$

❑ Total distance covered by the body before coming to rest $= h\left(\frac{1+e^2}{1-e^2}\right)$

(4) After the perfectly inelastic collision, the two bodies stick together. If m_1 and m_2 be the masses and v_1, v_2 be their velocities respectively then the velocity of the combination is given by $V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

and the loss of kinetic energy after the collision $\Delta E = \frac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$

(5) Stopping distance of the vehicle = $\frac{\text{Kinetic energy}}{\text{Stopping force}}$

(6) If a body moving with velocity v comes to rest after covering a distance x on a rough surface having coefficient of friction μ then $2\mu g x = v^2$

(7) Two vehicles of masses $m_1$and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same force, their stopping distance are x_1 and x_2 respectively.

$\frac{x_1}{x_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = \frac{E_1}{E_2}$ and their stopping times are t_1 and t_2 then $\frac{t_1}{t_2} = \frac{x_1 / v_1}{x_2 / v_2} = \frac{m_1 v_1}{m_2 v_2} = \frac{P_1}{P_2}$

(8) If the vehicles are moving with the same velocities and stopped by the same retarding force then

$\frac{x_1}{x_2} = \frac{m_1}{m_2}$ and $\frac{t_1}{t_2} = \frac{m_1}{m_2}$

(9) If the vehicles are moving with same kinetic energies and stopped by the same retarding force then

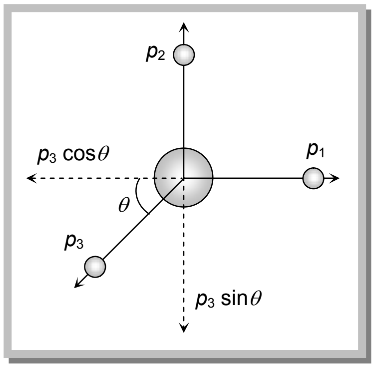
$\frac{x_1}{x_2} = 1$; $\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{\sqrt{2mE_1}}{\sqrt{2mE_2}} = \sqrt{\frac{E_1}{E_2}}$

(10) If the vehicle are moving with same linear momentum and stopped by the same retarding force then

$\frac{t_1}{t_2} = \frac{P_1}{P_2} = 1$ $\therefore t_1 = t_2$ but $\frac{x_1}{x_2} = \frac{E_1}{E_2} = \frac{m_2}{m_1}$

(11) A chain is lying on a friction less table such that its $(1/n)^{th}$ part is hanging at the edge. If the length of the chain is l and its mass is m, then work done in pulling up the hanging part will be $\frac{mgl}{2n^2}$.

(12) If a stationary body breaks due to some interaction in three parts out of which the first two parts move at right angles to each other with momenta p_1 and p_2 respectively, then the momentum of third part is determined as follows. According to law of conservation of momentum in horizontal direction.



$p_3 \cos\theta = p_1$ and in vertical direction $p_3 \sin\theta = p_2$

Magnitude of $p_3 = \sqrt{p_1^2 + p_2^2}$ and direction p_3

$\tan\theta = \frac{p_2}{p_1} \therefore \theta = \tan^{-1}\left(\frac{p_2}{p_1}\right)$

Direction of p_3 from the direction of motion of first part $\left[\pi + \tan^{-1}\left(\frac{p_2}{p_1}\right)\right]$

and direction of p_3 from the direction of motion of second part $\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{p_2}{p_1}\right)\right]$

(13) A pump motor is used to deliver water at a certain rate from a given pipe.

To get n times water, force must be increased n^2 times while power n^3 times.