PIPES AND CISTERNS

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2.9.3.3 PIPES AND CISTERNS

2.9.3.3.1 Basic Concepts

Inlet: A pipe connected with a tank (or a cistern or a reservoir) is called an Inlet, if it fills it.

Outlet: A pipe connected with a tank is called an outlet, if it empties it.

2.9.3.3.2 Basic Principles

- If a pipe can fill a tank in x hours, then the part filled in 1 hour $= \frac{1}{x}$.
- If a pipe can empty a tank in y hours, then the part of the full tank emptied in 1 hour $= \frac{1}{y}$.

2.9.3.3.3 Specific Cases and Formulae

- If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled in 1 hour, when both the pipes are opened = $\left(\frac{1}{x} - \frac{1}{y}\right)$.

  $\therefore$ Time taken to fill the tank, when both the pipes are opened = $\frac{xy}{y-x}$

Example: An electric pump can fill a tank in 3 hours. Because of a leak in the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Solution: Work done by the leak in 1 hour $= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}$.

$\therefore$ The leak will empty the tank in 21 hours.

- If a pipe can fill a tank in x hrs and another can fill the same tank in y hrs, then the net part filled in 1 hr, when both the pipes are opened = $\left(\frac{1}{x} + \frac{1}{y}\right)$.

  $\therefore$ Time taken to fill the tank = $\frac{xy}{y+x}$

Example: Two pipes P and Q can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Solution: Time take to fill the tank = $\frac{36\times45}{36+45}$= 20 hrs.

- If a pipe fills a tank in x hrs and another fills the same tank in y hrs, but a third one empties the full tank in z hrs, and all of them are opened together, the net part filled in 1 hr = $\left[\frac{1}{x} + \frac{1}{y} - \frac{1}{z}\right]$

  $\therefore$ Time taken to fill the tank = $\frac{xyz}{yz+xz-xy}$ hrs.

Example: Pipe P can fill a tank in 20 hours while pipe Q alone can fill it in 30 hours and pipe R can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

Solution: Required time = $\frac{20\times30\times40}{30\times40+20\times40-20\times30} = \frac{120}{7} = 17\frac{1}{7}$ hrs.

Example: Two pipes P and Q can fill a cistern in 1 hour and 75 minutes respectively. There is also an outlet R. if all the three pipes are opened together, the tank is full in 50 minutes. How much time will be taken by R to empty the full tank?

Solution: Work done by R in 1 min. $=\left(\frac{1}{60}+\frac{1}{75}-\frac{1}{50}\right)=\frac{3}{300}=\frac{1}{100}$.

$\therefore$ R can empty the full tank in 100 minutes.

- A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time taken by the leak to empty the tank = $\frac{xy}{y-x}$ hrs.

Example: A pipe can fill a tank in 15 hours. Due to a leak in the bottom, it is filled in 20 hours. If the tank is full, how much time will the leak take to empty it?

Solution: Required time = $\frac{15\times20}{20-15}$= 60 hrs.

Some More Examples:

Example: Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes?

Solution: Pipe B should be closed after $\left(1-\frac{18}{24}\right)\times 32 = 8$ min.

Example: Two pipes P and Q would fill a cistern in 24 hours and 32 hours respectively. If both pipes are opened together, find when the first pipe must be turned off so that the cistern may be just filled in 16 hours.

Solution: The first pipe should work for $\left(1-\frac{16}{32}\right)\times 24$ hrs. = 12 hrs.

Example: A tank has a leak which would empty it in 8 hrs. A tap is turned on which admits 6 liters a minutes into the tank, and it is now emptied in 12 hrs. How many liters does the tank hold?

Solution: The filler tap can fill the tank in $\frac{12\times8}{12-8}$ = 24 hrs.

$\therefore$ Capacity of tank = $24\times 60\times 6$ = 8640 litres.

Example: A cistern is normally filled in 8 hrs but takes two hrs longer to fill because of a leak in its bottom. If the cistern is full, then how long will the leak take to empty the full cistern?

Solution: The leak will empty in $\frac{8\times(8+2)}{2}$ = 40 hrs.

Example: A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Solution: Work done by the waste pipe in 1 minute

$=\frac{1}{20}-\left(\frac{1}{12}+\frac{1}{15}\right)=-\frac{1}{10}$ [- ve sign means emptying]

Waste pipe will empty the full cistern in 10 minutes.

Example: If 3 pumps working 8 hours a day can empty a tank in 2 days. In how many hours a day must 4 pumps work to empty the tank in 1 day?

Solution: $\text{Pumps}_1\times$ $\text{Days}_1\times \text{Hours}_1$ a day= $\text{Pumps}_2\times \text{Days}_2\times \text{Hours}_2$ a day

Or, $3\times2\times8 = 4\times1\times T_2$

or $T_2 = \frac{3\times2\times8}{4}$ = 12 hours a day