

HEAT AND THERMODYNAMICS

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1. HEAT AND THERMODYNAMICS

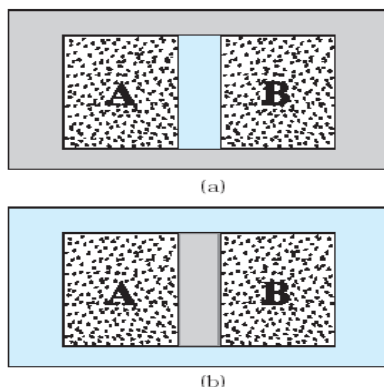
1.1 INTRODUCTION

Thermodynamics is the branch of physics which deals with the study of transformation of heat into other forms of energy and vice-versa.

1.2 THERMAL EQUILIBRIUM

Two objects are in thermal contact if they can affect each other's temperature. Thermal equilibrium exists when two objects in thermal contact no longer affect each other's temperature.

For example, if a carton of milk from the refrigerator is set on the kitchen countertop, the two objects are in thermal contact. After several hours, their temperatures are the same, and they are then in thermal equilibrium.



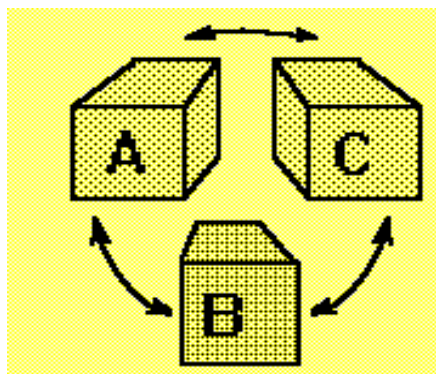
Consider two gases *A* and *B* occupying two different containers. We know experimentally that pressure and volume of a given mass of gas can be chosen to be its two independent variables.

Suppose first that the two systems are put in proximity but are separated by an adiabatic wall – an insulating wall (can be movable) that does not allow flow of energy (heat) from one to another. The systems are insulated from the rest of the surroundings also by similar adiabatic walls.

Next, suppose that the adiabatic wall is replaced by a diathermic wall – a conducting wall that allows energy flow (heat) from one to another. It is then found that the macroscopic variables of the systems *A* and *B* change spontaneously until both the systems attain equilibrium states. After that there is no change in their states

1.3 ZEROth LAW OF THERMODYNAMIC

This law states that if object **A** is in thermal equilibrium with object **B**, and object **B** is in thermal equilibrium with object **C**, then object **C** is also in thermal equilibrium with object **A**. This law allows us to build thermometers. For example the length of a mercury column (object **B**) may be used as a measure to compare the temperatures of the two other objects.



Zeroth law of thermodynamics corresponds to Existence of temperature of the body and thermal equilibrium.

1.3.1 Three different Scales of Temperature

- 1. The Celsius Scale:** This scale was devised by Anders Celsius in the year 1710. The interval between the lower fixed point and the upper fixed point is divided into 100 equal parts. Each division of the scale is called one degree centigrade or one degree Celsius (1°C). At normal pressure, the melting point of ice is 0°C . This is the lower fixed point of the Celsius scale. At normal pressure, the boiling point of water is 100°C . This is the upper fixed point of the Celsius scale.
- 2. The Fahrenheit Scale:** This scale was devised by Gabriel Fahrenheit in the year 1717. The interval into 180 equal parts. Each division of this scale is called one degree Fahrenheit (1°F). On this scale, the melting point of ice at normal pressure is 32°F . This is the lower fixed point. The boiling point of water at normal pressure is taken as 212°F . This is the upper fixed point.
- 3. The Reaumer Scale:** This scale was devised by R.A. Reaumer in the year 1730. The interval between the lower and the upper fixed points is divided into 80 equal parts. Each division is called one degree Reaumer (1°R). On this scale, the melting point of ice at normal pressure is 0°R . This is lower fixed point. The boiling point of water at normal pressure is 80°R . This is the upper fixed point.

1.3.2 Conversion of temperature from one scale to another

In order to convert temperature from one scale to another, following relation is used.

$$\frac{\text{Temperature on one scale-lower fixed point}}{\text{Upper fixed point-lower fixed point}} = \frac{\text{Temperature on one other side-lower fixed point}}{\text{Upper fixed point-lower fixed point}}$$

$$\therefore \frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{R-0}{80-0}$$

$$\frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

1.3.3 Constant volume Gas Thermometer

Suppose the pressure of the gas is p_0 when the bulb is placed in melting ice (ice point) and it is p_{100} when the bulb is placed in a steam bath (steam point). We assign 0°C to the temperature of the ice point and 100°C to the steam point. The temperature t corresponding to a pressure p of the gas is defined by

$$t = \frac{P - P_0}{P_{100} - P_0} \times 100^\circ\text{C}$$

1.3.4 Constant Pressure Thermometer

Volume of the bulb = V

Volume of the mercury taken out = v'

Temperature of the ice bath = T_0

Temperature of the heat bath = T

$$T = \frac{V}{V - v'} T_0$$

1.3.5 Platinum resistance Thermometer

Resistance at the temperature $t = R_t$

Resistance at the ice point = R_0

Resistance at the steam point = R_{100}

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

Electrical resistance of a metal wire increases gradually and uniformly over a fairly wide range of temperature has been made use of in electrical resistance thermometers. The variation of the resistance of a metal wire with temperature may be represented by the following approximate relation.

$$R_t = R_0 (1 + \alpha t)$$

Here, R_t is the resistance at $t^\circ\text{C}$, R_0 is the resistance at 0°C and α is the temperature coefficient of resistance. The value of α depends upon the nature of material of the wire.

1.4 THERMODYNAMIC STATE VARIABLES

In thermodynamics we define the state of a substance in terms of the various properties we can attribute to it.

1. Temperature T
 2. Pressure P
 3. Volume V
 4. Entropy S
-

5. Enthalpy H
6. Internal Energy Q
7. Mass m
8. Density ρ

Thermodynamic state variables of a system are the parameters which describe equilibrium states of the system

1.5 THERMODYNAMIC EQUATION OF STATE

The equation of state represents the connection between the state variables of a system.

For example, the equation of state of an ideal/perfect gas is represented as

$$PV = nRT$$

1.6 THERMODYNAMIC PROCESSES

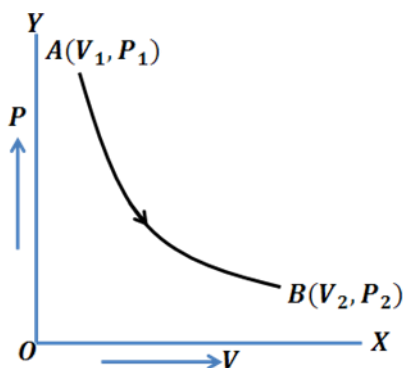
A system undergoes a thermodynamic process when there is some sort of energetic change within the system, generally associated with changes in pressure, volume, internal energy, temperature, or any sort of heat transfer.

The important thermodynamic processes are:

1. **Adiabatic process** - a process with no heat transfer into or out of the system.
2. **Isochoric process** - a process with no change in volume, in which case the system does not work.
3. **Isobaric process** - a process with no change in pressure.
4. **Isothermal process** - a process with no change in temperature.

1.7 INDICATOR DIAGRAM OR P-V DIAGRAM

The indicator diagram or P-V diagram represents the variation of volume V of a system with the pressure P of the system. The shape of the PV diagram shall depend upon the nature of thermodynamical process, the system undergoes. In figure, point A represents initial state and the point B represents final state of the system.

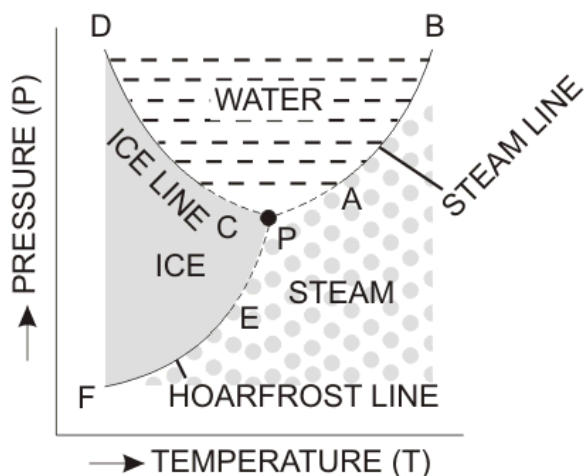


The areas under curves on the indicator diagram are equal to the work released by the process.

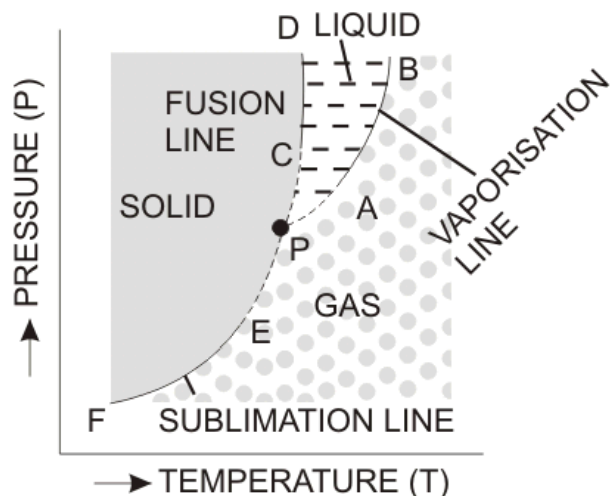
1.8 PRESSURE TEMPERATURE PHASE DIAGRAM

For three phase of matter, we need a three dimensional diagram in which pressure P, Volume V and temperature T should be plotted on three mutually perpendicular co-ordinate axes. As such, a PVT diagram is difficult to draw on a paper which is two dimensional. We usually ignore V and draw a graph between pressure P and temperature T. This graph is known as pressure temperature phase diagram. In such a diagram, each phase corresponds to a region or area. The boundary between any two areas is a curve on which the two phases co-exist in equilibrium.

PT diagram for water which contracts on melting (from ice into water)



PT diagram of substances like CO₂ which expand on melting



In each PT diagram, there are basically three curves:

- (i) **Vaporization line AB:** It represents the variation of boiling point of the substance (in liquid state) with the pressure. At any point on the vaporization line, the temperature and pressure values are such that the substance co-exists in equilibrium in liquid and vapour phase.

At pressure and temperature corresponding to any point above the vaporization line, the substance shall exist only in liquid phase. This is because at higher pressure all vapours will condense into liquid. Similarly, when pressure is decreases, liquid will evaporate into vapours i.e. at all points below the vaporization line, the substance shall exist only in the vapour state.

The vaporization line is called steam line in case of water

- (ii) **Fusion line CD:** It represents the variation of melting point of the substance (in solid form) with the pressure.

At every point on the fusion line, the temperature and pressure value are such that the substance co-exists in equilibrium in solid and the liquid phases.

The fusion line is called Ice line in case of water.

- (iii) **Sublimation line EF:** It represents the variation of pressure with temperature at which a solid changes directly into vapour state (without going into the liquid phase.)

Each point on sublimation curve gives us a set of values of P and T at which the substance coexists in equilibrium in the solid and the vapour phases.

The sublimation line is called the hoar frost line in case of water.

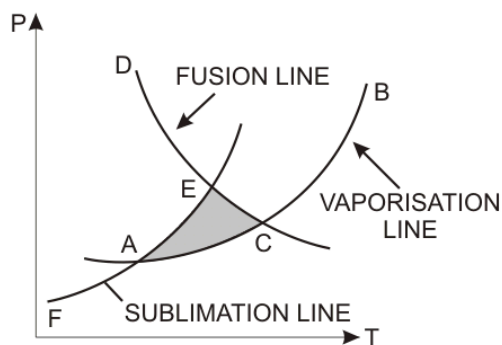
The three curves AB, CD and EF, when extrapolated, come to meet at a point P, which is called **Triple point** of the substance. 'Triple point' of a substance is a particular point on the P-T phase diagram representing a particular pressure and a particular temperature at which the substance can co-exist in all the three states viz solid, liquid and vapour.

The value of temperature and pressure corresponding to triple point are called triple point temperature and triple point pressure respectively.

For water, triple point temperature is 273.16°K or (0.01°C) and triple point pressure is 0.46 cm of Hg column.

For carbon dioxide, Triple point temperature is 216.4°K or -56.6°C and triple point pressure is 5.11 atmospheres.

NOTE: The three curves AB, CD and EF in PT phase diagram of a substance on extrapolation must meet at a single point P (which is the triple point of the substance).



If these curves were not to meet at O, they would enclose an area ACE as show in figure. As this area lies above the vaporization line AB, the substance must exist in liquid phase, at any point in area ACE.

Again, as the area ACE lies on the left of fusion line, the substance at any point in the area must exist in solid phase.

Further, as the area ACE lies on the right of sublimation line EF, the substance at any point in this area must exist in vapour phase.

Hence in the area ACE, the substance must exist in the three phases, solid, liquid and vapour which is impossible. Hence the vaporization line, fusion line and sublimation line must be meeting at a single point P, which is the triple point of the substance.

1.9 HEAT, INTERNAL ENERGY AND WORK

The total energy possessed by the system due to molecular motion (internal kinetic energy) and molecular configuration (internal potential energy) is called its internal energy. It is a macroscopic state variable of the system. It depends on state of the system. The kinetic energy of the container is not to be included in internal energy. It can be changed by changing any of the macroscopic state variables of the gas.

Heat and work in thermodynamics are not state variables. They are modes of energy transfer to a system resulting in change in its internal energy

Internal energy $U = PE + KE$ of molecules

KE of molecule = $(f/2)KT$

Where f = degree of freedom, K is Boltzmann constant.

PE of molecules depends upon intermolecular distance (r_0).

In case of ideal gases PE of molecules = 0. There by U depends on KE of gas molecules in turn depends on temperature.

1.10 FIRST LAW OF THERMODYNAMICS

The amount of heat supplied to a system (dQ) capable of doing external work is equal to sum of increase in the internal energy (dU) and external work done by system (dW).

$$dQ = dU + dW.$$

The first law of thermodynamics is the application of the conservation of energy principle to heat and thermodynamic processes.

Sign convention:

1. dQ is +ve when heat is supplied to the system and - ve when heat is rejected.
2. dU is +ve when temperature increases and -ve when temperature decreases.
3. dW is +ve when work is done by system and - ve when work is done on system.

1.10.1 Important points regarding first law of thermodynamics:

1. It is basically the law of conservation of energy.
 2. Applies equally to all the three phases of matter.
 3. All the three quantities dQ , dU and dW are expressed either in joule or in calorie.
 4. Introduces the concept of internal energy.
 5. It is impossible to get work from any machine, without giving an equivalent amount of energy to the machine.
 6. Establishes the essential equivalence between work and heat.
-

1.10.2 Applications of the First law of thermodynamics

1. Isothermal Process: In isothermal process temperature remains constant. For an ideal gas, internal energy remain constant i.e. $dU = 0$. The first law of thermodynamics implies

$$dQ = dU + dW = 0 + dW = dW$$

$$\text{i.e. } dQ = dW$$

i.e. heat supplied in an isothermal process is used entirely to do work against the external surroundings.

2. Adiabatic process: In this process no heat enters or leaves the system i.e. $dQ = 0$. The first law of thermodynamics implies

$$dQ = dU + dW = 0$$

$$\text{i.e. } dU = -dW$$

3. Isovolumic Process: In this process volume is kept constant i.e. $dW = P(dV) = 0$. The first law of thermodynamics implies

$$dQ = dU + dW$$

$$\text{i.e. } dQ = dU$$

i.e. heat absorbed by a system is equal to the change in internal energy of the system.

4. Cyclic process: In cyclic process the system is returned to its initial state after any number of changes. No intrinsic property of the system would change i.e. $dU = 0$. The first law of thermodynamics implies

$$dQ = dU + dW$$

$$\text{i.e. } dQ = dW$$

5. Melting Process: In this process internal energy increases.

Amount of heat absorbed during the melting, $dQ = mL$

Where m = mass of a solid

And L = latent heat of the solid

Change in volume is very small therefore can be neglected.

Therefore $dW = P(dV) = 0$.

According to the first law of thermodynamics

$$dQ = dU + dW \text{ or } mL = dU + 0$$

$$\text{Or } dU = mL$$

i.e. the internal energy during the melting process increases by mL .

6. Boiling Process: In this process liquid changes into vapour at constant temperature and pressure.

Suppose m = mass of a liquid at its boiling point

V_1 = volume of liquid

P = constant pressure at which boiling process occurs

V_2 = volume of same liquid in vapour form

L = latent heat of vaporization.

Therefore work done in expansion,

$$dW = P(dV) = P(V_2 - V_1)$$

Heat absorbed by the liquid in the boiling process

$$dQ = mL$$

According to first law of thermodynamics, $dQ = dU + dW$

Therefore $dU = mL - P(V_2 - V_1)$

1.11 SPECIFIC HEATS OF GASES

Amount of Heat required to raise the temperature of whole system by 1°C is Heat capacity of a gas (C)

$$C = \left[\frac{dQ}{dT} \right]$$

$C_v \rightarrow$ specific heat of a gas as constant volume.

$C_p \rightarrow$ specific heat of gas as constant pressure.

Units of C_p and C_v are J/kg/K

$C_p > C_v$

1.11.1 Relation between Specific heats of the gas

$$C_p - C_v = R$$

Where R is universal gas constant.

Ratio of specific heats

$$\frac{C_p}{C_v} = \gamma > 1$$

C_p & C_v in terms of R and γ

$$C_v = \frac{R}{\gamma - 1}, C_p = R \left[\frac{\gamma}{\gamma - 1} \right]$$

1.12 ISOTHERMAL PROCESS

Isothermal change is that change of pressure and volume when the temperature of the system remains constant. It is a slow process. It should be conducted in good thermally conducted vessel.

It follows Boyle's law. $P_1V_1 = P_2V_2$

During isothermal change, internal energy remains constant. U is constant

$$dU = 0$$

$$dQ = dW = PdV$$

Example: Melting of ice, boiling of a liquid are isothermal changes.

Work done in an isothermal change is given by

$$W = 2.303 nRT \log_{10} \left(\frac{V_2}{V_1} \right) = 2.303 nRT \log_{10} \left(\frac{P_1}{P_2} \right)$$

Work done depends on number of moles, Temperature and Expansion Ratio.

Isothermal elastic modulus = P

Slope of Isothermal curve

$$\left(\frac{dP}{dV} \right)_T = -\frac{P}{V}$$

Fractional change in pressure

$$\left(\frac{dP}{P} \right) = -\left(\frac{dV}{V} \right)$$

No two Isothermals intersect each other

1.13 ADIABATIC PROCESS

Adiabatic change is that change of pressure and volume during which heat is neither given to the system, nor taken from it. Temperature rises during adiabatic expansion. It is a quick process. Exchange of heat will not take place between system and surroundings. It should be conducted in perfectly bad conducting vessel.

$$\frac{C_p}{C_v} = \gamma$$

During adiabatic process, entropy remains constant. Hence it is also known as isentropic process.

In adiabatic process, $dQ = 0$

$$dU + dW = 0$$

$$dU = -dW$$

In an adiabatic process

$$PV^\gamma = \text{constant} \Rightarrow P_1V_1^\gamma = P_2V_2^\gamma$$

$$TV^{\gamma-1} = \text{constant} \Rightarrow T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{T^\gamma}{P^{\gamma-1}} = \text{constant} \Rightarrow \frac{T_1^\gamma}{P_1^{\gamma-1}} = \frac{T_2^\gamma}{P_2^{\gamma-1}}$$

Work done in an adiabatic process is given by

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{nR(T_2 - T_1)}{\gamma - 1}$$

Adiabatic Elastic Modulus = γP

Adiabatic Expansion of Gas is associated with decrease in pressure and temperature. Adiabatic compression of a gas associated with increase in pressure and temperature.

Slope of adiabatic curve

$$\left(\frac{\Delta P}{\Delta V}\right)_Q = -\frac{\gamma P}{V}$$

Slope of isothermal curve

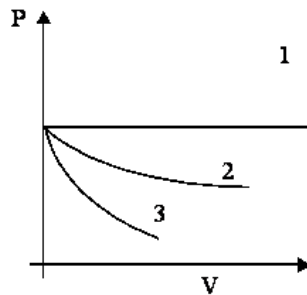
$$\left(\frac{\Delta P}{\Delta V}\right)_T = -\frac{P}{V}$$

$$\frac{\text{slope of adiabatic curve}}{\text{slope of isothermal curve}} = \gamma$$

Since $\gamma > 1$

\Rightarrow Slope of adiabatic curve $>$ slope of isothermal curve.

1.14 GRAPHS OF EXPANSION PROCESS

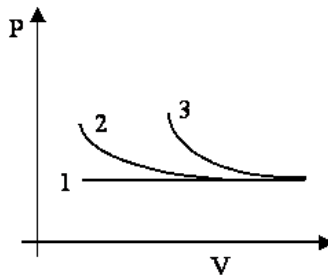


Line 1 represents isobaric process, curve 2 represents isothermal process and curve 3 represents adiabatic process.

Work done in isobaric process $>$ isothermal $>$ Adiabatic for same change in volume

For same increase in volume, starting from same point, final pressure in adiabatic process is less than that of in isothermal process.

1.15 GRAPHS OF COMPRESSION PROCESS



Curve 1 represents isobaric process, Curve 2 represents isothermal process and Curve 3 represents adiabatic process.

Work done in isobaric process > isothermal > Adiabatic process

Either in compression or expansion process, to produce equal change in volume of a gas, more pressure difference is required in adiabatic change.

1.16 SECOND LAW OF THERMODYNAMICS

Planck statement:

It is impossible to construct a heat engine which can completely convert heat energy into mechanical energy without rejecting heat to surroundings.

Kelvin statement:

It is impossible to extract work from a system by cooling it below surrounding temperature. It is impossible to transfer heat energy from body at lower temperature to body at higher temperature unaided by external agency.

1.17 REVERSIBLE AND IRREVERSIBLE PROCESS

(a) Reversible process:

Any process which can be made to proceed in reverse direction by variations in its conditions so that all changes occurring in the direct process are exactly reversed in the reverse process is called a reversible process.

(b) Irreversible process:

Any process which cannot be made to proceed in reverse direction is called an irreversible process. A part of the energy of the system performs work against dissipative forces and it cannot be recovered back. A few examples of irreversible processes are, diffusion of gases, rusting of iron, sudden expansion of a gas, work done against friction etc.

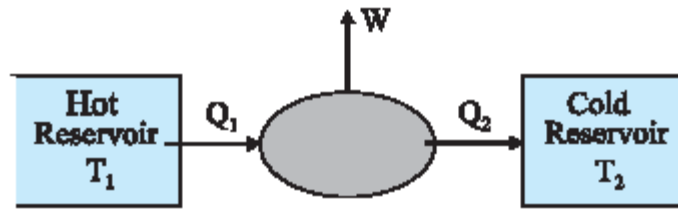
1.18 HEAT ENGINE

Device which converts heat energy into mechanical energy is called heat engine.

Parts of heat engine:

- Source of heat at higher temperature T_1
-

- Working substance
- Sink of heat at lower temperature T_2



Working:

- The working substance absorbs some heat Q_1 from the hot reservoir or the source.
- It then undergoes a cyclic change consisting of several processes.
- The working substance releases some heat energy Q_2 to cold reservoir or the sink

1.18.1 Thermal efficiency of a heat engine

The amount of energy input that is actually converted to useful output is called thermal efficiency of a heat engine. Efficiency is normally stated as a percentage:

$$\begin{aligned} \text{Efficiency} &= (\text{Energy out} / \text{Energy in}) \times 100\% \\ &= W / Q_1 \end{aligned}$$

According to the First Law of Thermodynamics, over one complete cycle

$$W = Q_1 - Q_2$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

As some heat is always rejected to the sink, $Q_2 \neq 0$. Therefore $\eta < 1$.

For $Q_2 = 0$, $\eta = 1 = 100\%$.

1.18.2 Types of Heat engines

In practice, heat engines are of two types:

1. External combustion engine and
2. Internal combustion engine

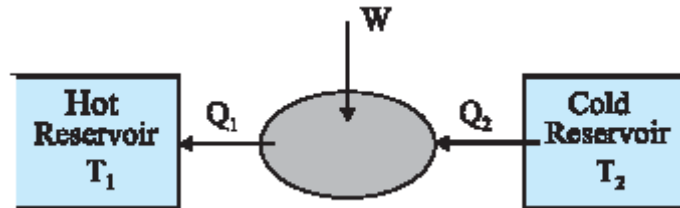
In external combustion engine heat is produced by burning the fuel in a chamber outside the main body of the engine. For example, steam engine.

In internal combustion engine heat is produced by burning the fuel in a chamber inside the main body of the engine. Petrol engine and diesel engine are internal combustion engines. They are also called four stroke engines.

1.18.3 Principle of a refrigerator (or heat pump)

A refrigerator is the reverse of a heat engine.

The working substance extracts heat Q_2 from the cold reservoir at temperature T_2 , some external work W is done on it and heat Q_1 is released to the hot reservoir at temperature T_1

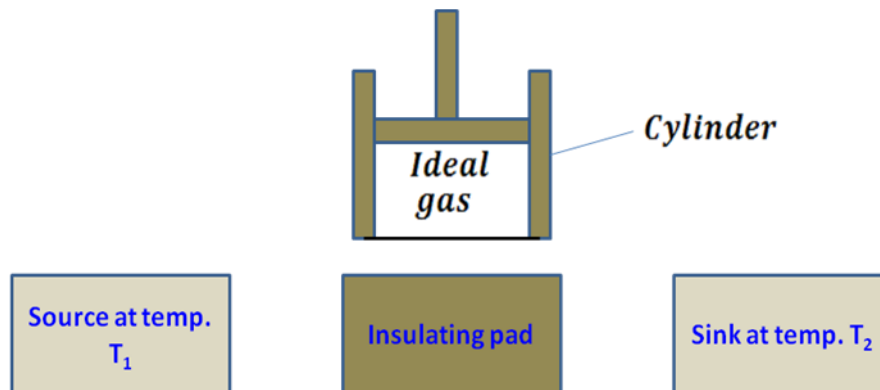


1.19 CARNOT CYCLE

The Carnot cycle is a particular thermodynamic cycle, modeled on the hypothetical Carnot heat engine, proposed by Sadi Carnot in 1824.

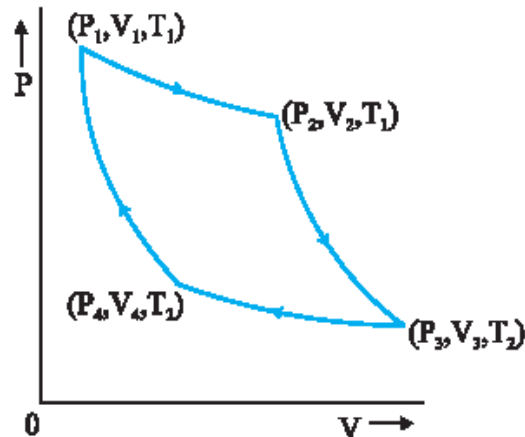
Construction:

- Source of heat:** maintained at a fixed higher temperature T_1 , from which the working substance draws heat.
- Sink of heat:** maintained at a fixed lower temperature T_2 , to which any amount of heat can be rejected by the working substance.
- Working substance:** A perfect gas contained in a cylinder with non-conducting sides but having a perfectly conducting-base.



Stages of Carnot cycle

- Isothermal expansion
 - Adiabatic expansion
 - Isothermal compression
 - Adiabatic compression
-



- (a) *Step 1* → 2 Isothermal expansion of the gas taking its state from (P_1, V_1, T_1) to (P_2, V_2, T_1) . The heat absorbed by the gas (Q_1) from the reservoir at temperature T_1 is given by

$$Q_1 = W_1 = \int_{V_1}^{V_2} P.dV = RT_1 \log_e \frac{V_2}{V_1}$$

- (b) *Step 2* → 3 Adiabatic expansion of the gas from (P_2, V_2, T_1) to (P_3, V_3, T_2) . Work done by the gas is

$$W_2 = \int_{V_2}^{V_3} P.dV = \frac{R(T_2 - T_1)}{(1 - \gamma)}$$

- (c) *Step 3* → 4 Isothermal compression of the gas from (P_3, V_3, T_2) to (P_4, V_4, T_2) . Heat released (Q_2) by the gas to the reservoir at temperature T_2 is given by

$$Q_2 = W_3 = \int_{V_3}^{V_4} -P.dV = RT_2 \log_e \frac{V_3}{V_4}$$

- (d) *Step 4* → 1 Adiabatic compression of the gas from (P_4, V_4, T_2) to (P_1, V_1, T_1) . Work done on the gas is

$$W_4 = \int_{V_4}^{V_1} -P.dV = \frac{-R(T_2 - T_1)}{(1 - \gamma)}$$

Total work done by the gas in one complete cycle is

$$W = W_1 + W_2 - (W_3 + W_4) = W_1 - W_3 \quad [\text{As } W_2 = W_4]$$

$$W = Q_1 - Q_2 = \text{Area of loop}$$

1.19.1 Efficiency of Carnot engine

The amount of energy input that is actually converted to useful output. Efficiency is normally stated as a percentage:

$$\begin{aligned} \text{Efficiency} &= (\text{Energy out/ Energy in}) \times 100\% \\ &= W/Q_1 \end{aligned}$$

According to the First Law of Thermodynamics, over one complete cycle

$$W = Q_1 - Q_2$$
$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$
$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$
$$\eta = 1 - \frac{T_2}{T_1}$$

1.19.2 Carnot theorem

According to this theorem

- Working between two given temperatures, T_1 of hot reservoir (the source) and T_2 of cold reservoir (the sink), no engine can have efficiency more than that of the Carnot engine.
- The efficiency of the Carnot engine is independent of the nature of the working substance.

1.20 ENTROPY

For a closed thermodynamic system, a quantitative measure of the amount of thermal energy not available to do work is called entropy. In other words entropy is a measure of the disorder or randomness in a closed system.

On a macroscopic scale, entropy is a function of thermodynamic variables, as temperature, pressure, or composition, that is a measure of the energy that is not available for work during a thermodynamic process. A closed system evolves toward a state of maximum entropy.

1.21 SOLVED EXAMPLES

Example 1: An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960 \text{ J}$, $Q_2 = -5585 \text{ J}$, $Q_3 = -2980 \text{ J}$ and $Q_4 = 3645 \text{ J}$ respectively.

The corresponding quantities of work involved are $W_1 = 200 \text{ J}$, $W_2 = -825 \text{ J}$, $W_3 = -1100 \text{ J}$ and W_4 respectively.

Find the value of W_4 . What is the efficiency of the cycle?

Solution: As the process is cyclic, therefore, $\Delta U = 0$

According to first law of thermodynamics $\Delta Q = \Delta U + \Delta W = \Delta W$

$$\text{i. e., } \Delta W = \Delta Q \text{ or } W_1 + W_2 + W_3 + W_4 = Q_1 + Q_2 + Q_3 + Q_4$$

$$W_4 = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$$

$$= (5960 - 5585 - 2980 + 3645) - (200 - 825 - 1100)$$

$$\text{Efficiency} = \frac{\text{Network done}}{\text{total heat absorbed}} = \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}$$

$$\eta = \frac{2200 - 825 - 1100 + 765}{5960 + 3645} = \frac{1040}{9605} = 0.1083$$

$$\eta = 0.1083 \times 100\% = 10.83\%$$

Example 2: The temperature T_1 and T_2 of two heat reservoirs in an ideal Carnot engine are 1500°C and 500°C . Which of these increasing T_1 by 100°C or decreasing T_2 by 100°C , would result in greater improvement of the efficiency of the engine?

Solution: Using

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

i) Increasing T_1 by 100°C

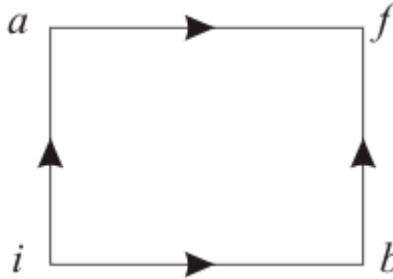
$$\eta_1 = \frac{1600 - 500}{(1600 + 273)} = \frac{1100}{1873} \approx 59\%$$

ii) Decreasing T_2 by 100°C

$$\eta_2 = \frac{1500 - (500 - 100)}{(1500 + 273)} = \frac{1100}{1773} \approx 67\%$$

Clearly, decreasing T_2 by 100°C results in greater improvement of efficiency

Example 3: When a system is taken from state i to state f along the path iaf , it is found that $Q = 50 \text{ Cal}$ and $W = 20 \text{ Cal}$, fig. Along the path ibf , $Q = 36 \text{ Cal}$. What is the value of W along the ibf ?



Solution: Along the path iaf ,
 Increase in internal energy,
 $U_1 = Q_1 - W_1 = 50 - 20 = 30 \text{ Cal}$
 Along the path ibf ,
 Increase in internal energy, $U_2 = Q_2 - W_2$
 $U_2 = 36 - W$
 As $U_2 = U_1 \quad \therefore 36 - W = 30$
 $W = 36 - 30 = 6 \text{ Cal}$.

Example 4: A Carnot engine having an efficiency of $\eta = \frac{1}{10}$ as heat engine is used as a refrigerator. If the work done on the system is 10 J , what is the amount of energy absorbed from the reservoir at lower temperature?

Solution: Here, $\eta = \frac{1}{10}, W = 10 \text{ J}, Q_2 = ?$
 Coefficient of performance of refrigerator
 $C.O.P = \frac{1-\eta}{\eta} = \frac{Q_2}{W}$
 $\frac{1-\frac{1}{10}}{\frac{1}{10}} = \frac{Q_2}{10}$
 $\therefore Q_2 = 90 \text{ J}$

Example 5: If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

- $C_p - C_v = 28R$
- $C_p - C_v = R/28$

c. $C_p - C_v = R/14$

d. $C_p - C_v = R$

Solution: For nitrogen, $M = 28$

From $C_p - C_v = \frac{R}{M}$

Where symbols have their usual meaning,

$$C_p - C_v = R/28$$

1.22 HEAT TRANSFER

1.22.1 INTRODUCTION

Heat Transfer helps us to understand various aspects for the transfer of heat through various materials (solids, liquid and gases) through various known process such as conduction, convection, radiation etc.

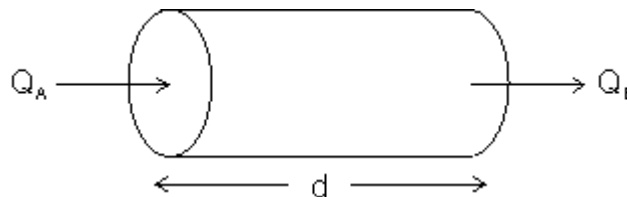
Heat is transferred from higher temperature to lower temperature.

There are three different ways in which heat can be transferred:

1. **Conduction:** it is process by which heat can be transferred in solids.
2. **Convection:** it is a process by which heat is transferred in fluids (gaseous & liquid)
3. **Radiation:** it is a process in which the heat is transferred in the form of electromagnetic waves without the aid of any material medium.

1.22.2 THERMAL CONDUCTIVITY

In solids, heat is transferred through conduction. We will study conduction of heat through a solid bar in the following sector.



Consider a solid bar of thickness d and area of cross-section A . The left side of bar is maintained at Q_A temperature and right side at Q_B . Let us suppose heat flows from higher temperature to lower temperature after some time, the temperature of each section becomes constant with time. This is known as steady state.

In steady state, if ΔQ Amount of heat crosses through any cross section in time ΔT , then

$$\frac{\Delta Q}{\Delta T} \propto A \quad \text{and} \quad \frac{\Delta Q}{\Delta T} \propto \frac{Q_A - Q_B}{d}$$

$$\frac{\Delta Q}{\Delta T} = \frac{KA(Q_A - Q_B)}{d}$$

K is coefficient of thermal conductivity.

For a small thickness dx along the direction of heat and whose thickness dx is small and temperature difference is ΔQ

$$\frac{\Delta Q}{\Delta T} = \frac{dQ}{dt} = \frac{-KAdQ}{dx}$$

Quantity dQ/dx is called the temperature gradient and minus sign indicates that dQ/dx is negative along the direction of heat flow.

Question: Pieces of iron and glass are heated to same temperature. Which one will be hotter? Why?

Answer: Pieces of iron will be hotter because of higher thermal conductivity.

Question: Cooking utensils are provided with wooden handles why?

Answer: Wood is poor conductor of heat. So hot utensils can be easily handled.

Question: In winter, metallic handles of wooden door appear colder. Why?

Answer: Because they are good conductor of heat.

Question: Why is ice packed in gunny bags or sawdust?

Answer: Air trapped in saw dust prevents transfer of heat from the surroundings, to the ice.

1.22.3 ABSORPTIVE POWER

Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body.

$$\text{Absorptive power } a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

1.22.4 EMISSIVE POWER E

The emissive power denotes the energy radiated per unit time per unit area of the surface.

1.22.4.1 Emissivity

The emissivity of a surface is the ratio of the Emissive power of the surface to the emissive power of the black body at that temp.

$$E = \frac{\text{Emmissive Power of the surface}}{\text{Emmissive Power of the Black Body}}$$

1.22.5 BLACK BODY

A perfectly black body is the one which absorbs completely all the radiations of whatever wavelength incident on it.

1.22.6 KIRCHOFF'S LAW

It states that the ratio of the Emissive power to the Absorptive power for radiation of a given wavelength is the same for all bodies at the same temperature.

$$\frac{e_{\lambda}}{a_{\lambda}} = E_{\lambda}$$

1.22.7 STEFAN'S LAW OF RADIATION

The total radiant energy emitted E per unit time by a black body of surface A is proportional to the fourth power of its absolute temperature.

$$E \propto T^4$$

$$\text{or } E = \sigma AT^4$$

Where σ = Stefan's const

For a body which is not black body

$$E = \epsilon\sigma AT^4$$

Where ϵ = emissivity of the Black Body.

Note: Emmissitivity and absorptive power have the same value.

Net loss of thermal Energy

If a body of surface area A is kept at absolute temp T in a surrounding of temperature T_0 ($T_0 > T$) Then the energy emitted by the body per unit time.

$$E = \epsilon\sigma AT^4$$

and the energy absorbed per unit time by the body

$$E_0 = \epsilon\sigma AT_0^4$$

Net loss of thermal energy per unit time.

$$\Delta E = E - E_0 = \epsilon\sigma A [T^4 - T_0^4]$$

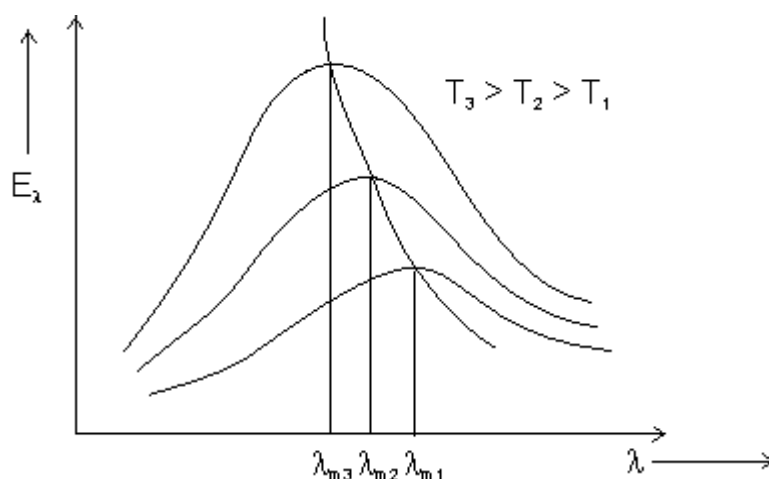
1.22.8 NEWTON'S LAW OF COOLING

For a small temperature difference between a body and its surroundings, the rate of cooling of the body is directly proportional to the temperature difference. If a body of temperature T and surface area A is kept in a surrounding temperature T_0 ($T_0 < T$), then net loss of thermal energy per unit time.

$$dQ/dt = \epsilon\sigma A (T - T_0)$$

1.22.9 WIEN'S LAW

At a constant temperature T , when wavelength λ is increased, the energy emitted E , first increases, reaches a maximum and then decreases i.e at a particular temperature the spectral radiancy E_{λ} is a maximum at a particular wavelength λ_m .



As the temperature increases, the maximum radiancy of energy occurs at shorter wavelength i.e

$$\lambda_m T = b$$

b is a constant.

This is called Wien's Displacement Law.

1.22.10 SOLAR CONSTANT

The solar constant is defined as the amount of heat energy received per second per unit area by a perfect black body placed at the surface of the Earth with its surface being held perpendicular to the direction of the sun's rays.

The value of solar constant is 1388 Wm^{-2} or $2 \text{ cal cm}^{-2} \text{ min}^{-1}$.

The instrument used for measuring the solar constant is called Pyroheliometer.

1.22.10.1 Surface temperature of the sun

Let R be the radius of the sun and r be the radius of earth's orbit around the sun. Let E be the energy emitted by the sun per second per unit area. Then, the total energy emitted by the sun is one second $= 4\pi R^2 \times E$. This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the sun i.e., on a sphere of surface area $4\pi r^2$.

$$\text{The energy falling per unit area} = \frac{4\pi R^2 \times E}{4\pi r^2} = \frac{E \times R^2}{r^2}$$

By definition, this is the solar constant S

$$\text{i.e., } S = \frac{ER^2}{r^2} \text{ but } E = \sigma T^4$$

$$\text{According to Stefan's law, } S = \frac{\sigma T^4 R^2}{r^2} \text{ or } T^4 = \frac{S r^2}{\sigma R^2}$$

$$\text{Or } T = \left[\frac{S r^2}{\sigma R^2} \right]^{1/4}$$

Now $S = 1388 \text{ Wm}^{-2}$, $R = 6.96 \times 10^8 \text{ m}$, $r = 1.496 \times 10^{11} \text{ m}$,

$$\sigma = 5.68 \times 10^{-8} \text{ SI units}$$

On substituting these values above, we get T , the surface temperature of the sun. It is found to be equal to 5791 K.

1.22.11 SOLVED EXAMPLES

Example 1: Temperatures of two stars are in ratio 3:2. If wavelength of maximum intensity of first body is 4000 \AA , what is corresponding wavelength of second body?

Solution: According to Wien's displacement law

$$\lambda_m T = \text{constant}$$

$$\therefore \frac{(\lambda_m)_1}{(\lambda_m)_2} = \frac{T_2}{T_1}$$

$$\text{Here, } \frac{T_1}{T_2} = \frac{3}{2}, (\lambda_m)_1 = 4000 \text{ \AA} = 4000 \times 10^{-10} \text{ m}$$

$$\therefore (\lambda_m)_2 = \frac{4000 \times 10^{-10} \times 3}{2}$$

$$= 6000 \text{ \AA}$$

Example 2: The luminosity of the Rigel star is 17000 times that of the sun. Assume both to be perfectly black bodies. If the surface temperature of the sun is 6000 K, What is the temperature of the star?

Solution: Luminosity of a star depends upon the total radiations emitted by the star.

The star emits 17000 times the radiations emitted by the sun

$$E = \sigma T^4$$

$$\text{Hence, } \frac{E_1}{E} = \left(\frac{T_1}{T}\right)^4$$

$$\text{So, } (17000)^{1/4} = \frac{T_1}{T} \quad (\text{Given, } E_1 = 17000 E)$$

$$T_1 = 6000 \times 11.4 = 68400 \text{ K}$$

Example 3: According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surrounding and n is equal to

(a) 2

(b) 3

(c) 4

(d) 1

Solution: According to Newton's law of cooling

$$\frac{dQ}{dt} \propto \Delta\theta$$

$$\text{But } \frac{dQ}{dt} \propto (\Delta\theta)^n \quad (\text{given})$$

$$\therefore n = 1$$