EXPONENTIAL AND LOGARITHMIC SERIES

EXPONENTIAL AND LOGARITHMIC SERIES

- **Exponential series:** Consider the following infinite series of numbers

$$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!} \qquad \ldots(\text{i})$$

The sum of this series is denoted by the symbol *e*.

Observations:

✓ The value of e may be computed to any desired number of decimal places by using series.

✓ The number that is not algebraic is called *transcendental*. The class of transcendental numbers includes π and e.

- ✓ For any base e, we have
  $\log_e 1 = 0$ since $e^0 = 1$ and $\log_e e = 1$ since $e^1 = e$.
  Therefore, e is a number whose logarithm is 1

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.. + \frac{x^n}{n!} + \qquad \ldots(ii)$$

On replacing x by –x, we get

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots.. \text{and so } e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots..$$

This series is known as an exponential series.

- **Some Particular Cases**

- ✓ For x = 1, we get $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots..$

- For x = -1, we get $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...$
- $\frac{1}{2}(e + e^{-1}) = 1 + \frac{1}{3!} + \frac{1}{5!} + ...$
- $\frac{1}{2}(e - e^{-1}) = 1 + \frac{1}{2!} + \frac{1}{4!} + ...$
- $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...$
- $\frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} - \frac{x^4}{4!} + ...$
- $\frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} - \frac{x^5}{5!} + ...$

- **Logarithmic series:** For any real x,

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots \qquad \ldots.(i)$$

The series on the right hand side of (i) is called the logarithmic series which holds if |x|< 1.

- **Some Particular Cases:**

✓ $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots.$

✓ $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots.$

Which is obtained by putting x = 1 in (i) and is called Leibnitz series.

✓ $\log \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots. \right)$

- **Exponential and Logarithmic Functions**

  Definition 1. The exponential function f with base a > 0, $a \neq 1$, is defined by $f(x) = e^{x^{\log_e a}}$ for every real number x. It is denoted by a^x.

  Definition 2. The inverse of the exponential function with base a is called the logarithmic function with base a and is denoted by $\log_a$.

  Since $y = f^{-1}(x)$ if and only if $x = f(y)$, the definition of $\log_a$ may be expressed as follows:

  $y = \log_a x$ if and only if $x = a^x$.

  $a^{\log_a x} = x$ for every $x > 0$, $\log_a a = 1$ and $\log_a 1 = 0$

  Observations:

- ✓ Let $y_1 = \log_e x_1$, $y_2 = \log_e x_2$ so that $x_1 = e^{y_1}$, $x_2 = e^{y_2}$. then, $y_1 + y_2 = \log_e x_1 + \log_e x_2 = \log_e x_1 x_2$.

Thus, we have

$$e^{y_1+y_2} = e^{\log_e x_1 x_2} = x_1 x_2 = e^{y_1} \times e^{y_2}$$

$$\vec{r} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$$

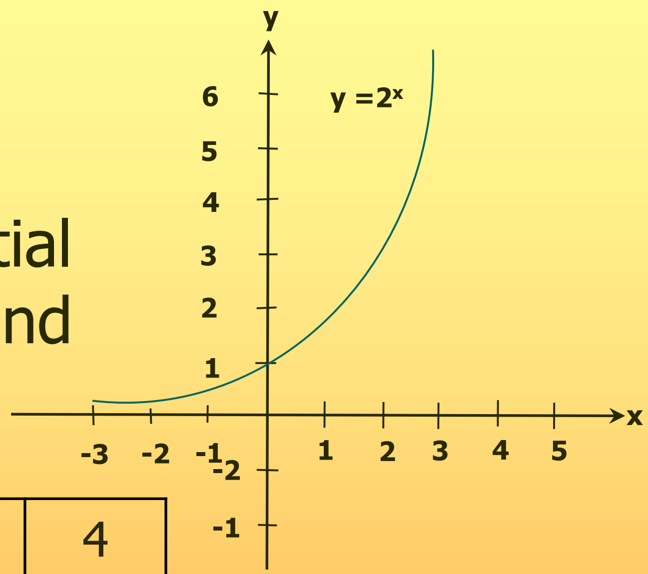
- ✓ In particular, if we take a = e, then y = e^x if and only if x = $\log_e$ y.
- ✓ Since f(x) = e^x and g(x) = log x are inverse functions, they are symmetric with respect to the line y = x.
- ✓ If a > 0 and x is a real number, then denoting by log the natural logarithm

$$a^x = e^{x\log a} = 1 + \frac{\log a}{1!}x + \frac{(\log a)^2}{2!}x^2 + \ldots \text{.is an expansion of } a^x.$$

- **Graph of Exponential Functions**

✓ **Graph of** $y = 2^x$

Since here $a = 2 > 1$, the exponential function is increasing throughout R and $y = 2^x > 0$ for all x.



X	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

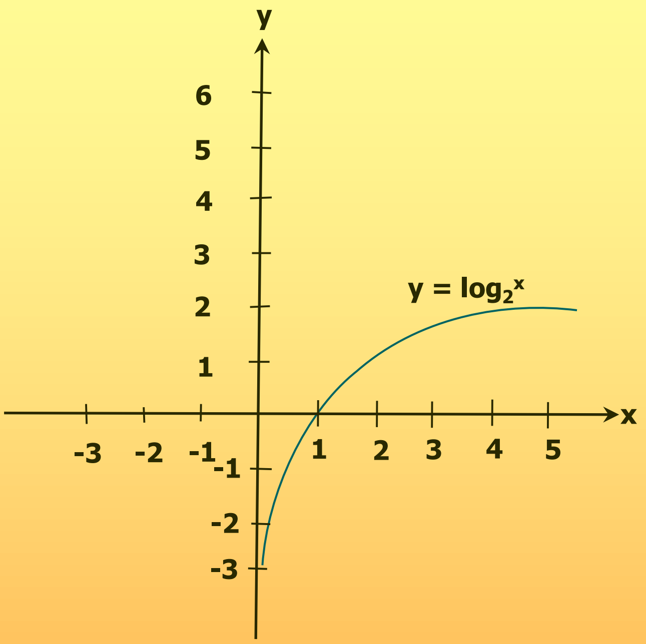
Since $a > 1$, 2^x decreases as x decreases through negative values. It approaches the x-axis but never intersects it. However, its y-intercept is 1. The graph of $y = 2^x$ is drawn above.

- **Graph of Logarithmic Functions**

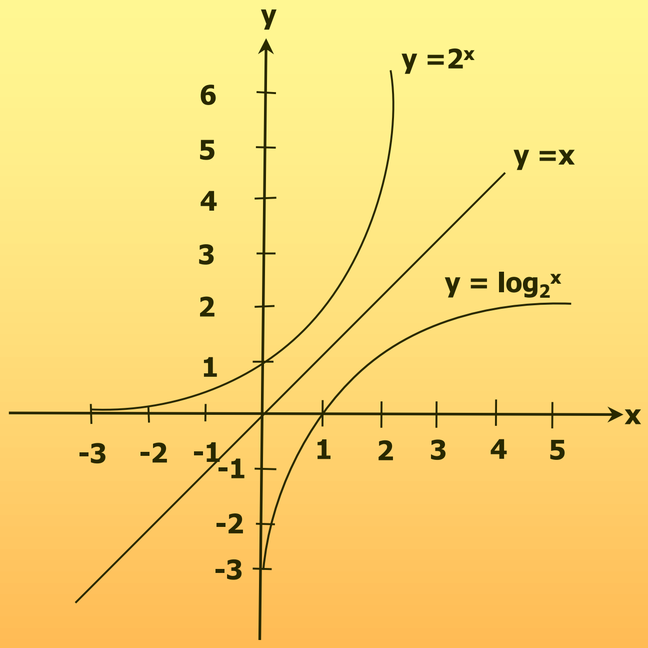
  Graph of $y = \log_2 x$

  Since the logarithmic function $\log_a$ is the inverse of the exponential function with base *a*, the graph of $y = \log_a x$ can be obtained by reflecting the graph of $y = a^x$ on the line $y = x$. In the case of $y = \log_2 x$, the coordinates of the points on the graph may be found by using the equation $x = 2^y$.

Y	-3	-2	-1	0	1	2	3
x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Since the logarithmic function is defined for only $x > 0$, the graph of $y = \log_2^x$ exists only for these x. The graph intersects x-axis at (1, 0).



Since $y = 2^x$ and $y = \log_2 x$ are inverse function, their graphs are symmetric with respect to the line $y = x$ as shown in Fig.

Thank You…