

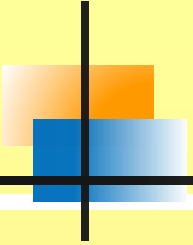


PERMUTATIONS AND COMBINATIONS



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- **Fundamental Principle of counting (FPC):** If an event can occur in m different ways, following which another event can occur in r different ways and so forth, then the total number of different ways of occurrence of the events in the given order is $m \times n \times r \times \dots$.
This principle is also called *Multiplication Principle*.
- **Combinations:** A group of r or all things chosen out of the given n things is called a *combination*. (Here the order is not taken into account).
The number of combinations of r things out of the given n things is denoted by ${}^n C_r$ or ${}^n C_r$ or $C(n, r)$.

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- **Permutations:** A group of r or all things chosen out of the given n things and arranged in a definite order is called a *permutation*.

The number of permutation of r things out of the given n things is denoted by ${}^n P_r$ or ${}^n P_r$ or $P(n, r)$.

In general, if we select r objects from n distant objects, then each combination of r objects can be permuted in $r!$ ways. Thus, relation between ${}^n C_r$ and ${}^n P_r$ is given by

$${}^n C_r \times r! = {}^n P_r, \text{ i.e., } {}^n C_r = \frac{{}^n P_r}{r!}$$

Using the formula ${}^n P_r = \frac{n!}{(n-r)!}$, we obtain ${}^n C_r = \frac{n!}{r!(n-r)!}$



- **Fundamental Theorems**

- ✓ **Addition Theorem:** If an operation can be performed in m ways and another operation can be performed in n ways independent of the first operation then either of the two operations can be performed in $(m + n)$ ways.

- ✓ **Multiplication Theorem:** If an operation can be performed in m ways and after it has been performed in any one of these ways, a second operation can be performed in n ways, then both these operations can be performed in $m \times n$ ways.

- **Factorial Notation:** The product of first n natural numbers is written as $\angle n$ or $n !$ and pronounced as factorial n .

Thus, $n! = 1.2.3\dots n = (n-1)! \times n$

Also $0! = 1$



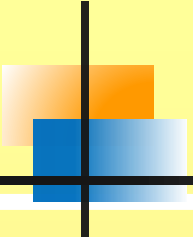
- **Important Results:**

- ✓ The number of permutations of r things out of n different things is given by

$${}^n P_r = n(n-1)(n-2)\dots(n-r+1) = \frac{n!}{(n-r)!}$$

where $n! = 1.2.3\dots(n-1) n = n(n-1)!$ and $0! = 1$.

- ✓ The number of permutation of n different things taken all at a time is ${}^n P_n$ or $n!$.
- ✓ The number of permutations of n different things taken r at a time, when the repetition is allowed, is n^r .
- ✓ The number of permutation of n things taken all at a time when p things are of one kind, q are of second kind, r are of third kind and the rest are all different, is $\frac{n!}{p!q!r!}$



✓ ${}^n P_r = {}^{n-1} P_r + r \cdot {}^{n-1} P_{r-1}$.

✓ The number of combinations of n different things r at a time is given by

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

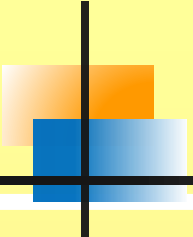
✓ The number of combinations of n things in which at least one thing is selected is $2^n - 1$.

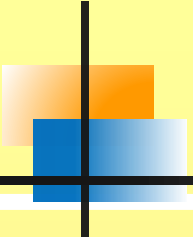
✓ The number of combinations of $(p + q + r)$ things when p alike are of one kind, q are alike of second kind, r are alike of third kind, is

$$(p - 1) (q - 1) (r - 1) - 1$$

✓ ${}^n C_{n-r} = {}^n C_r$

✓ ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$ for $r < n$

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- ✓ Number of *combinations* of n distinct things taken r at a time when p - particular things always occur $= {}^{(n-p)}C_{(r-p)}$.
 - ✓ The number of permutations of n distinct things taken r at a time when p - particular things always occur $= {}^{(n-p)}C_{(r-p)} \cdot r!$
 - ✓ Number of combinations of n distinct things taken r at a time when p - particular things never occur $= {}^{(n-p)}C_r$.
 - ✓ Number of permutations (arrangements) of n distinct things taken r at a time when p particular things never occur $= {}^{(n-p)}C_r r!$
 - ✓ If some or all of n things be taken at a time, then the number of combinations $= {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n - 1$.



✓ Number of ways to make a selection by taking some or all of $p_1 + p_2 + \dots + p_r$ things, where p_1 are alike of one kind, p_2 are alike of one kind, p_r are alike of r th kind, is given by $[(p_1 + 1)(p_2 + 1)\dots(p_r + 1) - 1]$.

✓ If there are p_1 objects of one kind, p_2 objects of second kind, ..., p_n objects of n th kind, then the number of ways of choosing r objects out of these $(p_1 + p_2 + \dots + p_n)$ objects = coefficient of x^r in

$$(1 + x + \dots + x^{p_1})(1 + x + \dots + x^{p_2})(1 + x + \dots + x^{p_n}).$$

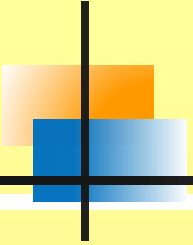
If one object of each kind is to be included in such a collection, then the number of ways of choosing r objects = coeff. of x^r in the product.

$$(x + x^2 + \dots + x^{p_1})(x + x^2 + \dots + x^{p_2})(x + x^2 + \dots + x^{p_n}).$$



✓ **Circular Arrangement**

- The number of ways of permuting n distinct objects along a circle is $(n-1)!$
- The number of ways of arranging n distinct objects along a circle when clockwise and anti-clockwise arrangements are considered alike is $\frac{1}{2}(n-1)!$
- The number of ways of arranging n persons along a round table so that no person has the same neighbours is $\frac{1}{2}(n-1)!$
- The number of necklaces formed with n beads of different colours (or garlands) with n different flowers is $\frac{1}{2}(n-1)!$

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- The number of ways of dividing $3m$ different objects into three groups of m objects each is

✓ $\frac{(3m)!}{(m!)^3}$, when distinction can be made between the three groups.

✓ $\frac{(3m)!}{(m!)^3 3!}$, when no distinction can be made between the groups.



Thank You...