PERMUTATIONS AND COMBINATIONS

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- **Fundamental Principle of counting (FPC):** If an event can occur in *m* different ways, following which another event can occur in r different ways and so forth, then the total number of different ways of occurrence of the events in the given order is $m \times n \times r \times \ldots$.
  This principle is also called *Multiplication Principle*.

- **Combinations:** A group of *r* or all things chosen out of the given *n* things is called a *combination*. (Here the order is not taken into account).
  The number of combinations of *r* things out of the given *n* things is denoted by ^nCr or ^nC_r or C(n, r).

- **Permutations:** A group of r or all things chosen out of the given *n* things and arranged in a definite order is called a *permutation.*
  The number of permutation of *r* things out of the given *n* things is denoted by ^nP_r or ^nPr or P (n, r).

  In general, if we select *r* objects from n distant objects, then each combination of *r* objects can be permuted in r! ways. Thus, relation between ^nC_r and ^nP_r is given by

$$^nC_r \times r! = {}^nP_r, i.e., {}^nC_r = \frac{^nP_r}{r!}$$

Using the formula $^nP_r = \dfrac{n!}{(n-!)}$, we obtain $^nC_r = \dfrac{n!}{r!(n-r)!}$

- **Fundamental Theorems**

✓ **Addition Theorem:** If an operation can be performed in m ways and another operation can be performed in n ways independent of the first operation then either of the two operations can be performed in (m + n) ways.

✓ **Multiplication Theorem:** If an operation can be performed in m ways and after it has been performed in any one of these ways, a second operation can be performed in n ways, then both these operations can be performed in $m \times n$ ways.

- **Factorial Notation:** The product of first n natural numbers is written as $\angle n$ or n ! and pronounced as factorial n.

Thus, $n! = 1.2.3\ldots n = (n-1)! \times n$

Also $0! = 1$

- **Important Results:**

✓ The number of permutations of r things out of n different things is given by

$$^{n}P_{r} = n(n-1)(n-2)....(n-r+1) = \frac{n!}{(n-!)}$$

where n! = 1.2.3….(n-1) n = n(n-1)! and 0! = 1.

✓ The number of permutation of n different things taken all at a time is $^{n}P_{n}$ or n!.

✓ The number of permutations of n different things taken r at a time, when the repetition is allowed, is n^{r}.

✓ The number of permutation of n things taken all at a time when p things are of one kind, q are of second kind, r are of third kind and the rest are all different, is $\frac{n!}{p!q!r!}$

- ✓ $^{n}P_{r} = {}^{n-1}P_{r} + r.\ {}^{n-1}P_{r-1}$.
- ✓ The number of combinations of n different things r at a time is given by

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

- ✓ The number of combinations of n things in which at least one thing is selected is $2^{n} - 1$.
- ✓ The number of combinations of (p + q + r) things when p alike are of one kind, q are alike of second kind, r are alike of third kind, is
  (p - 1) (q - 1) (r - 1)-1
- ✓ $^{n}C_{n-r} = {}^{n}C_{r}$
- ✓ $^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$ for $r < n$

- Number of *combinations* of *n* distinct things taken r at a time when p- particular things always occur = ${}^{(n-p)}C_{(r-p)}$.
- The number of permutations of n distinct things taken r at a time when p- particular things always occur = ${}^{(n-p)}C_{(r-p)}$. r !
- Number of combinations of n distinct things taken r at a time when p- particular things never occur = ${}^{(n-p)}C_r$.
- Number of permutations (arrangements) of n distinct things taken r at a time when p particular things never occur = ${}^{(n-p)}C_r\, r!$
- If some or all of n things be taken at a time, then the number of combinations = ${}^nC_1 + {}^nC_2 + \ldots. + {}^nC_n = 2^n - 1$.

- ✓ Number of ways to make a selection by taking some or all of $p_1 + p_2 + \ldots + p_r$ things, where p_1 are alike of one kind, p_2 are alike of one kind, p_r are alike of rth kind, is given by $[(p_1 + 1)(p_2 + 1)\ldots(p_r + 1)-1]$.

- ✓ If there are p_1 objects of one kind, p_2 objects of second kind, …., p_n objects of nth kind, then the number of ways of choosing r objects out of these $(p_1 + p_2 + \ldots + p_n)$ objects = coefficient of x^r in $(1 + x + \ldots + x^{P_1})(1 + x + \ldots + x^{P_2})(1 + x + \ldots + x^{P_n})$.
  If one object of each kind is to be included in such a collection, then the number of ways of choosing r objects = coeff. of x^r in the product.
  $(x + x^2 \ldots + x^{P_1})\ (x + x^2 \ldots + x^{P_2})\ (x + x^2 \ldots + x^{P_n})$.

✓ **Circular Arrangement**

- The number of ways of permuting n distinct objects along a circle is (n-1)!
- The number of ways of arranging n distinct objects along a circle when clockwise and anti-clockwise arrangements are considered alike is $\frac{1}{2}(n-1)!$
- The number of ways of arranging n persons along a round table so that no person has the same neighbours is $\frac{1}{2}(n-1)!$
- The number of necklaces formed with n beads of different colours (or garlands) with n different flowers is $\frac{1}{2}(n-1)!$

- The number of ways of dividing 3m different objects into three groups of m objects each is

✓ $\frac{(3m)!}{(m!)^3}$, when distinction can be made between the three groups.

✓ $\frac{(3m)!}{(m!)^3 3!}$, when no distinction can be made between the groups.

Thank You…