Elasticity

Some Important Definition

(1) **Deforming force :** A deforming force is one which when applied changes the configuration (either in length, volume or shape) of the body.

(2) **Elasticity :** The property of the body to regain its original configuration (length, volume or shape) when the deforming forces are removed is called elasticity.

(3) **Perfectly elastic body :** A body which regains its original configuration immediately and after. The removed of forming force from it is called perfectly elastic body.

Example : Quartz and phosphor bronze are the example of nearly perfectly elastic bodies.

(4) **Perfectly plastic body :** A body which does not regain its original configuration at on the removed of deforming force, howsoever small the deforming. Force may be is called perfectly plastic body

Example : Putty and paraffin wax are the example of the nearly perfectly plastic bodies.

(5) **Stress :** The restoring force per unit area set up inside the body is called stress and is measured by the magnitude of the deforming force acting on unit area of the body when the equilibrium is established.

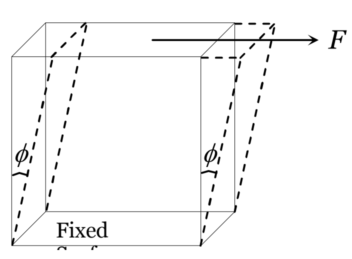
$$\text{Stress} = \frac{\text{External deforming force}}{\text{Area}}$$

Units : $\frac{\text{Newton}}{m^2}$ in S.I. and $\frac{dyne}{cm^2}$ in C.G.S; Dimension = $\left[ML^{-1}T^{-2}\right]$

Types : (i) Normal stress : The internal restoring force setup per unit area of the body is called normal stress normal stress can be classified into two types –

(ii) **Tensile stress :** If there is an increase in the length or extension of the body in the direction of the force applied the stress setup is called tensile stress.

(iii) **Compression stress :** If there is decrease in length of the wire or compression of the body due to force applied, the stress setup is called compression stress.



(iv) **Tangential stress :** When a deforming force, acting tangentially to the surface of a body produces a change in the shape of the body, then the stress setup in the body is called tangential stress.

If a tangential force F applied on the top face of cubical body (of surface area a)

Then tangential stress = $\frac{F}{a}$

(6) **Strain :** The ratio of change in configuration to the original configuration is called strain *i.e.*

Strain = $\frac{\text{change in configuration}}{\text{original configuration}}$. Being the ratio of two like quantities, it has no dimensions and units.

Type of strain :

(i) Longitudinal strain $= \dfrac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$

(ii) Volumetric strain $= \dfrac{\text{change in volume}(\Delta V)}{\text{original volume}(V)}$

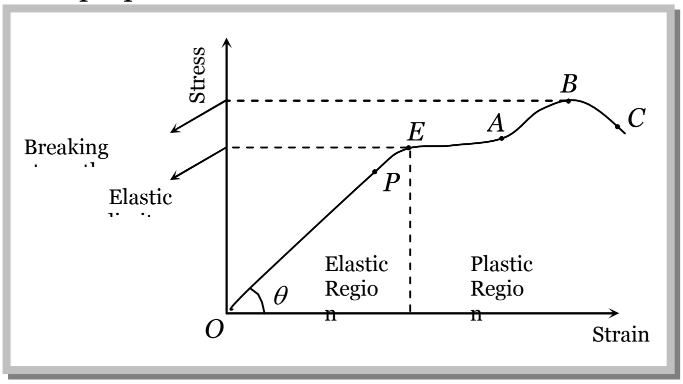
(7) **Shearing strain :** It is defined as angle in radian through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

Stress Strain Curve

When a wire is suspend vertically from a rigid support and at it lower end load is increased continuously then we get the curve between stress (or load) and strain (or elongation) as shown in the figure.

Important point of the graph :

(1) When the strain is small (<2 %) (in region *OP*) stress is proportional to stream. Hook's law is followed in this region. The point P is called limit of proportionality and slope of line *OP* gives the young's modulus *Y* of the material of the wire. $Y = \tan\theta$



(2) If the strain is further increased *i.e.* in the region *PE* , the stress is not proportional to strain. However the wire still regain its original length after the removal of stretching force. This behaviour is shown up to point E known as elastic limit or yield-point. The region *OPE* represent the elastic behaviour of the material of the wire.

(3) If the wire is stretched beyond the elastic limit E, *i.e.* between *EA*, the strain increases much more rapidly and if the stretching force is removed the wire does not come back to its natural length. Some permanent increase in length takes place.

(4) If the stress is increased further, by very small increase in it a very large increase in strain is produced (region *A* and after reaching point *B*, the strain increases even if the wire is unloaded and rupture at *C*. In the region *BC* the wire literally flows. The maximum stress correspond to *B* after which the wire begins to flow and breaks is called breaking or tensile strength. The region *EABC* represent the elastic behavior of the material of wire.

(5) If the plastic region between *E* and *B* large the material is said to be ductile and can be drawn in to wires and if it is small, the material is said to be brittle as it will break soon after the elastic limit is crossed.

Hooke's Law and Moduli of Elasticity

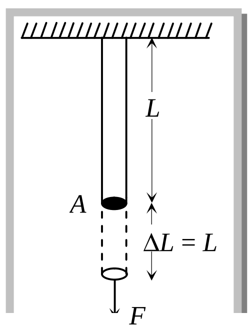
(1) **Hooke's Law :** Within elastic limit, the stress developed is directly proportional to the strain produced in the body

i.e. Stress $\propto$ Strain or $\dfrac{\text{Stress}}{\text{Strain}} = E = \text{constant}$

Here this constant of proportional to E is known as modulus of elasticity.

Depending on the type of stress applied and resulting strain the modulus of the elasticity are of three types :

(i) **Young's Modulus (*Y*) :** It is defined as the ratio of normal stress to the longitudinal strain with in the elastic limit

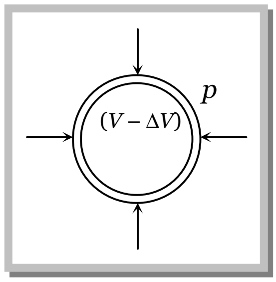


$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{Mg/\pi r^2}{l/L} = \frac{MgL}{\pi r^2 l}$$

Units : $\frac{Newton}{m^2}$ or Pascal in S.I. and $\frac{dyne}{cm^2}$ in C.G.S.; Dimension : $[ML^{-1}T^{-2}]$

(ii) **Bulk Modulus :** It is defined as the ratio of normal stress to the volumetric strain, with in the elastic limit.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}} = \frac{F/A}{\Delta V/V} = \frac{-pV}{\Delta V}$$

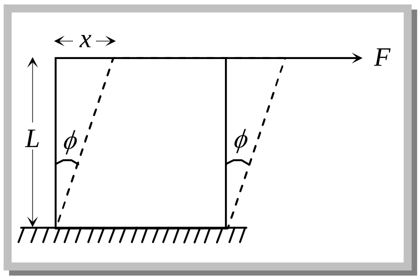


where p = increase in pressure; V = original volume; ΔV = change in volume

The negative sign shows that with increase in pressure P, the volume decreases by ΔV *i.e.* if P is positive, ΔV is negative.

The reciprocal of bulk modulus is called compressibility $C = \text{compressibility} = \frac{1}{k} = \frac{\Delta V}{PV}$

All the states of matter possess volume elasticity. Bulk modulus of gases is very low while that of liquids and solids is very high.



(iii) **Modulus of rigidity (η):** It is defined as the ratio of tangential stress to the shearing strain with in the elastic limit. It is also called shear modulus of rigidity.

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\phi} = \frac{F/A}{x/L} = \frac{FL}{xA}$$

Some Important Points About Moduli of Elasticity

(1) The value of moduli of elasticity (Y, K and η) is independent of the magnitude of the stress and strain. It depends only on the nature of the material of the body.

(2) For a given material there can be different moduli of elasticity depending on the type of stress applied and resulting strain.

(3) The moduli of elasticity has same dimensional formula and units as that of stress since strain is dimension less.

(4) Greater the value of moduli of elasticity, more elastic is the material. But as $\gamma \propto \frac{1}{l}, K \propto \frac{1}{\Delta V}$ and $\eta \propto \frac{1}{\phi}$ for a constant stress, so smaller change of shape or size for a given stress corresponds to greater elasticity.

(5) Y and η exists only for solids as liquids and gases cannot be deformed along one dimension only and also cannot sustain shear strain. However K exists for all states of matter like solid, liquid or gas.

(6) Bulk modulus of gases is very low while that for liquids and solids is very high $E_{solid} > E_{liquid} > E_{gas}$

(7) Gases have two bulk –moduli, namely isothermal elasticity E_θ. It has been found that at given pressure P

$$E_\theta = P \text{ and } E_\phi = \gamma P \quad \text{So that } \frac{E_\phi}{E_\theta} = \gamma = \frac{C_P}{C_V} > 1 \qquad i.e.\ E_\phi > E_\theta$$

It means adiabatic elasticity is γ times greater than isothermal elasticity.

(8) With rise in temperature, Y, K and η decreases.

Poisson Ratio

Within the elastic limit, the ratio of lateral strain to longitudinal strain is constant for a given material and is known as Poisson ratio. $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$. Theoretical value $\rightarrow -1 < \sigma < 1/2$,

Practical value $\rightarrow 0 < \sigma < 1/2$

Relation between Y, K, η and σ: $Y = 3K(1-2\sigma)$; $Y = 2\eta(1+\sigma)$; $Y = \frac{9\eta K}{3K+\eta}$; $\sigma = \frac{3K-2\eta}{6K+2\eta}$

Applications

(1) If the length of a wire is doubled, the longitudinal strain will be $\frac{\Delta L}{L} = \frac{L_2 - L_1}{L_1} = \frac{2L - L}{L} = 1$

But as $Y = \frac{\text{Stress}}{\text{Strain}}$, $Y = \text{Stress}$ [as strain $= 1$].

So young's modulus is numerically equal to the stress which will double the length of a wire.

(2) **For a loaded wire :** $l = \frac{FL}{\pi r^2 Y}$ as $\left[Y = \frac{F}{A}\frac{L}{l} \text{ and } A = \pi r^2\right]$

If same stretching force is applied to different wires of same material $l \propto \frac{L}{r^2}$ [as F and Y = constant].

That means greater the ratio $\frac{L}{r^2}$ greater will be the elongation

(3) **Increment in length of wire by its own weight :** If wire of length L and cross section A is stretched by a force F, by definition of young's modulus $l = \frac{FL}{AY}$

In case of elongation by its own weight, $F(= Mg)$ will act at centre of gravity of the wire, so that length of wire which is stretched will be $(L/2)$

$$l = \frac{MgL/2}{AY} = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y} \quad [\text{as } M = \rho AL]$$

(4) **Thermal Stress :** If a rod is fixed between two supports due to change in temperature its length will change and so it will exert a normal stress (compressive if temperature increases and tensile if temperature decreases) on the supports. This stress is called thermal stress.

Thermal stress = $Y\alpha\Delta\theta$

(5) **Work done in stretching a wire :** When a wire stretched, some work is done against the internal restoring forces acting between the particles of the wire. This work done appears in the form of potential energy of the wire.

$$U = \frac{1}{2}Fl = \frac{1}{2}\frac{F}{A}\cdot\frac{l}{L}(AL) = \frac{1}{2}\cdot stress\cdot strain\cdot volume \qquad \left[\begin{array}{l} F = \text{force; } l = \text{elongation ; } L = \text{length of wire;} \\ A = \text{cross - section area; } V = \text{volume} \end{array}\right.$$

$$\Rightarrow \text{ Energy per unit volume} = \frac{1}{2}\cdot stress\cdot strain = \frac{1}{2}\cdot Y\cdot(strain)^2 = \frac{1}{2}\frac{(stress)^2}{Y} \qquad \left[\because As\;\; Y = \frac{stress}{strain}\right]$$

(6) In case of bending of a beam of length L, breadth b and thickness d, by a load Mg at the middle.

Depression δ is given by $\delta = \dfrac{MgL^3}{4Ybd^3}$

(7) Twisting of a cylinder (or wire) of length L and radius r, elastic restoring couple per unit twist is given by

$$C = \pi\eta r^2/2L$$

(8) Work done in twisting the cylinder through an angle $\theta \Rightarrow W = \dfrac{1}{2}C\theta^2$

(9) Rod of length L and radius r fixed at one end angle of shear ϕ is related to angle of twist θ by the relation

$$L\phi = r\theta$$

Elastic After Effect

When the deforming force is removed the body tend to return to their respective original states some body returns immediately and others takes appreciably long time to do so.

This delay in regaining the original state by a body after the removed of deforming force is called elastic after effect.

Quartz and Phosphor-bronze shows the negligible elastic after effect but glass fibre takes hours to return to its original state.

Elastic Fatigue

It is defined as the loss of strength the material caused due to repeated alternating strains to which the material is subjected. And due to elastic fatigue its behavior corresponds to that of less elastic bodies. Elastic body relieved of the fatigue or regains its original degree of elasticity, when allowed to rest for some time.

Breaking Stress

The stress by applying which the wire breaks is called breaking stress.

For a given material like modulus of elasticity, Breaking Stress is constant. It does not depends on the dimension of wire.

But breaking force depends upon the thickness of the wire.

Breaking force = Breaking stress × Area of cross section of the wire

Breaking force ∝ Area of cross section of wire. Breaking stress = constant.

Factor of safety : While using a material, the working stress is always kept much lower than that of breaking stress. So safety factor is the ratio of breaking stress and that of working stress.

$$\text{Safety factor} = \frac{\text{breaking stress}}{\text{working stress}}$$

Inter Atomic Force Constant (K)

It is equal to the product of young's modulus of the material of the wire Y and normal distance r_0 between the atoms of the wire $K = Y \times r_0$.

As well as $K = \dfrac{\text{interatomi c force}}{\text{interatomi c distance}} = \dfrac{F}{\Delta r}$; Unit $= \dfrac{\text{newton}}{\text{meter}}$

Note : ❑ **When the wire break by its own weight then the length of wire**

As breaking Force = Breaking stress $\times$ Area of cross section

$Mg = P \times A \Rightarrow Mg = P \times M / L \Rightarrow P = Ldg$ L = length of wire, d = density of wire

Breaking stress = Ldg or $L = \dfrac{P}{dg}$

❑ If the length of metallic wire is L_1 when the tension in the wire is T_1 and is L_2 when the tension is T_2 then the original length of wire $L = \dfrac{L_1 T_2 - L_2 T_1}{T_2 - T_1}$.