PERCENTAGE

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3.2 Percentage

3.2.1 Definition

A **percentage** is a way of expressing a number as a fraction of 100 (*per cent* meaning "per hundred"). It is often denoted using the percent sign, "%". For example, 35% (read as "thirty-five percent") is equal to 35 / 100, or 0.35.

Percentages are used to express how large/small one quantity is, relative to another quantity. The first quantity usually represents a part of, or a change in, the second quantity, which should be greater than zero. For example, an increase of Rs. 0.15 on a price of Rs. 2.50 is an increase by a fraction of 0.15 / 2.50 = 0.06. Expressed as a percentage, this is therefore a 6% increase.

Although percentages are usually used to express numbers between zero and one, any dimensionless proportionality can be expressed as a percentage. For instance, 222% is 2.22 and -0.35% is -0.0035.

3.2.2 Conversion between Percent, Fraction and Decimal

- A decimal number is converted to a percentage by multiplying the decimal number by 100, or by moving the decimal point two places to the right.
- A percentage can be converted into decimal form by dividing it by 100, or by moving the decimal point two places to the left.
- A fraction can be converted into a percentage by changing the fraction to a decimal form and then changing the decimal to a percentage.
- A percentage can be changed into a fraction by first converting the percentage into a decimal form and then changing the decimal form to a fraction. For example.
  1% = 0.01 = 1/100, 100% = 1.0 = 1
  2% = 0.02 = 2/100, 120% =1.2 = 12/10

Example: What percentage is equivalent to $\frac{3}{8}$?

Solution: $\frac{3}{8} \times 100 = \frac{75}{2}$=37.5%

Example: $16\frac{2}{3}\%$ of 600gm$-33\frac{1}{3}\%$ of 180gm

Solution: $= \left[\left(\frac{50}{3} \times \frac{1}{100} \times 600\right) - \left(\frac{100}{3} \times \frac{1}{100} \times 180\right) \right]$ gm = (100 - 60) gm = 40 gm.

Example: $\frac{1}{2}$ is what percent of $\frac{1}{3}$?

Solution: Required percentage $= \left(\frac{1}{2} \times \frac{3}{1} \times 100\right)\% = 150\%$

3.2.3 Specific Cases of Percentage

3.2.3.1 Percent relationship between quantities

- If two values are respectively x % and y % more than a third value, then the first is of the $\frac{100+x}{100+y} \times 100\%$

Example: Two numbers are respectively 20% and 50% more than a third. What is the first as % of the second?

Solution: The required value $= \frac{120}{150} \times 100 = 80\%$

- If A is x % of C and B is y% of C, then A is $\frac{x}{y}\times 100\%$ of B.

Example: Two numbers are respectively 20% and 25% of a third number. What percentage is the first of second?

Solution: The required value $= \frac{20}{25}\times 100 = 80\%$

- x % of a quantity is taken by the first, y % of the remaining is taken by the second and z% of the remaining is taken by third person. Now, if A is the quantity left, then original quantity was:

$$\frac{A\times 100\times 100\times 100}{(100-x)(100-y)(100-z)}$$

Example: 3.5% of income is taken as tax and 12.5% of the remaining is saved. This leaves Rs. 4,053 to spend. What is the income?

Solution: $\text{Income} = \frac{4053\times 100\times 100}{(100-3.5)(100-12.5)} = \text{Rs } 4{,}800$

Example: After deducting 10% from a certain sum, and then 20% of the remainder, there is Rs 7,200 left. Find the original sum.

Solution: The required sum $= \frac{7200\times 100\times 100}{90\times 80} = \text{Rs } 10{,}000$

- x % of a quantity is added. Again, y % of the increased quantity is added. Again z% of the increased quantity is added. Now, it becomes A, then the initial quantity is given by

$$\frac{A\times 100\times 100\times 100}{(100+x)\ (100+y)\ (100+z)}$$

Example: A man had Rs 2,400 in his locker two years ago. In the first year, he deposited 20% of the amount in his locker. In the second year, he deposited 25% of the increased amount in his locker. Find the amount at present in his locker.

Solution: The required amount $= \frac{2400\times 120\times 125}{100\times 100} = \text{Rs } 3{,}600$

- If first value is r% more than the second value, then the second is less than the first value by $[\frac{r}{(100+r)}\times 100]\%$

Example: If A's salary is 25% more than that of B, then how much percent is B's salary less than that of A?

Solution: When A's salary is Rs 100, B's salary is $\frac{25}{125}\times 100 = \text{Rs } 20$ less than that of A i.e., B's salary is 20% less than that of A.

- If the first value is r% less than the second value then, the second value is more than the first value by $(\frac{r}{(100-r)}\times 100)\%$

Example: If A's salary is 30% less than that of B, then how much percent is B's salary more than that of A?

Solution: B's salary more than that of A by $= \frac{30}{100-30}\times 100 = 42\frac{6}{7}\%$

When the value is first increased and then decreased:

- If the value of a number is first increased by x% and later decreased by x %, the net change is always a decrease which is equal to x % of x or $\frac{x^2}{100}$.

Example: A shopkeeper marks the price of his goods 12% higher than its original price. After that, he allows a discount of 12%. What is his percentage profit or loss?

Solution: % value of loss $= \frac{(12)^2}{100} = 1.44\%$

When both values are different:

- If the value is first increased by x % and then decreased by y % then there is increase or decrease, $(x - y - \frac{xy}{100})\%$ according to the +ve or −ve sign respectively.

Example: The price of tea being increased by 20%, a man reduces his consumption by 20%. By how much percent will his expenses for tea be decreased?

Solution: % expense = (20 – 20 - $\frac{20\times20}{100}$) % = - 4% (decreases because sign is negative).

- If the order of increase and decrease is changed, the result remains unaffected.

$$\text{Effect} = \%\ \text{increase} - \%\ \text{decrease} - \frac{\%\ \text{increase}\times\%\ \text{decrease}}{100}$$

Example: A shopkeeper marks the prices of his goods at 20% higher than the original price. After that, he allows a discount of 10%. What profit or loss did he get?

Solution: %Effect $= 20 - 10 - \frac{20\times10}{100} = 8\%$

He gets 8% profit as the sign obtained is +ve.

Successive increase or decrease:

- If the value is increased successively by x % and y % then the final increase is given by $[x + y + \frac{xy}{100}]\%$

Example: A shopkeeper marks the prices at 15% higher than the original price. Due to increase in demand, he further increases the price by 10%. How much % profit will he get?

Solution: % profit $= 15 + 10 + \frac{15\times10}{100} = 26.5\%$

3.2.3.2 Examinations and marks

- The pass marks in an examination is x %. If a candidate who secures y marks fails by z marks. Then the maximum marks,

$$\text{M} = \frac{100(y+z)}{x}$$

Example: A student has to secure 40% marks to get through. If he gets 40 marks and fails by 40 marks, find the maximum marks for the examination.

Solution: Maximum marks $= \frac{100(40+40)}{40} = 200$

- A candidate scoring x % in an examination fails by 'a' marks, while another candidate who scores y % marks gets 'b' marks more than the minimum required pass marks. Then the maximum marks for the examination are

$$= \frac{100(a+b)}{y-x}$$

Example: A candidate scores 25% and fails by 30 marks, while another candidate who scores 50% marks, gets 20 marks more than the minimum required marks to pass the examination. Find the maximum marks for the examination.

Solution: Maximum marks $= \frac{100(30+20)}{50-25} = 200$

- In an examination, x % failed in English and y % failed in Maths. If z % of students failed in both subjects, the percentage of students who passed in both the subjects is 100–(x + y + z) or, (100 – x) + (100 – y) + z.

Example: In an examination, 40% of the students failed in Maths, 30% failed in English and 10% failed in both. Find the percentage of students who passed in both the subjects.

Solution: The required % $= 100 - (40 + 30 - 10) = 40\%$

3.2.3.3 Geometry

- In measuring the sides of a rectangle, one side is taken x % in excess and the other y % in deficit. The error per cent in area calculated from the measurement is $x - y - \frac{xy}{100}$ in excess or deficit, according to the +ve or –ve sign.

Example: In measuring the sides of a rectangle, one side is taken 5% in excess and the other 4% in deficit. Find the error per cent in area calculated from the measurement.

Solution: % error= $5 - 4 - \frac{5 \times 4}{100} = 1 - \frac{1}{5} = \frac{4}{5}\%$ excess because sign is +ve

- If the sides of a triangle, rectangle, square, circle (it's radius) or rhombus (or any 2-dimentional figure) are increased by x %, its area is increased by

$$\frac{x(x+200)}{100}\% \text{ or } \left[2x + \frac{x^2}{100}\right]\%$$

Example: If each side of a square is increased by 25%, find the percentage change in its area.

Solution: Increase % $= \frac{25(25+200)}{100}\% = 56 \cdot 25\%$

3.2.3.4 Depreciation

- If the present value of a machine be Q, and it depreciates at the rate of r% per annum, Then

  (a) Value of machine after n years = $Q(1 - \frac{r}{100})^n$

  (b) Value of machine n years ago = $\frac{Q}{\left(1-\frac{r}{100}\right)^n}$

Example: The value of a machine depreciates at the rate of 10% per annum. If its present value is Rs. 1,62,000, what will be its worth after 2 years? What was the value of the machine 2 years ago?

Solution: Value of the machine after 2 years

$$= \text{Rs.}\left[162000 \times (1 - \frac{10}{100})^2\right] = \text{Rs.}\left(162000 \times \frac{9}{10} \times \frac{9}{10}\right) = \text{Rs.}131220.$$

Value of the machine 2 years ago

$$= \text{Rs.}\left[\frac{162000}{\left(1-\frac{10}{100}\right)^2}\right] = \text{Rs.}(162000 \times \frac{10}{9} \times \frac{10}{9}) = \text{Rs.}200000.$$

3.2.3.5 Population growth

- If the original population of a town is P, and the annual increase is r%, then

  (i) The population after n years = $P(1 + \frac{r}{100})^n$

Example: If the annual increase in the population of a town is 4% and the present number of people is 15,625, what will the population be in 3 years?

Solution: The required population $= 15625(1+\frac{4}{100})^3$

$= 15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = 17576$

(ii) The Population n years ago = $\frac{P}{\left(1+\frac{r}{100}\right)^n}$

Example: The population of a town increases by 5% every year. If the present population of the town is 138915, find its population 3 years ago.

Solution: The required population= $\frac{138915}{\left(1+\frac{5}{100}\right)^3}$

= 120000.

- If the original population of a town is P, and the annual decrease be r%, then the population in n years = $P(1-\frac{r}{100})^n$

Example: If the annual decrease in the population of a town is 5% and the present number of people is 1,62,000, what will the population be in 2 years?

Solution: The required population

$= 162000(1-\frac{5}{100})^2$

= 146205.

When the Rate of Growth is Different for Different Years:

- The population of a town is P. It increases by x% during the first year, increases by y % during the second year and again increases by z% during the third year. The population after 3 years will be

$= \frac{P \times (100+x)(100+y)(100+z)}{100 \times 100 \times 100}$

Example: The population of a town is 8000. It increases by 10% during the first year and by 20% during the second year. What is the population after two years?

Solution: The required population= $\frac{8000 \times 110 \times 120}{100 \times 100} = 10,560$

When Population increases for one Year and Then Decreases for the Next Year:

- If the population increases by x% during first year, decreases by y% during the second year and increases by z% during third year, the population after 3 years will be

$= \frac{P \times (100+x)(100-y)(100+z)}{100 \times 100 \times 100}$

Example: The population of a town is 10,000. It increases by 10% during the first year, it decreases by 20% during the second year and increased by 30% during the third year. What is the population after 3 years?

Solution: The required population

$= \frac{10000 \times 110 \times 80 \times 130}{100 \times 100 \times 100} = 11440$

Example: The population of a town increases at the rate of 10% during one year and it decreases at the rate of 10% during the second year. If it has 29,700 inhabitants at present, find the number of inhabitants two year ago.

Solution: The required population= $\frac{29700 \times 100 \times 100}{(100-10) \times (100+10)}$

$$= \frac{29700 \times 100 \times 100}{90 \times 110} = 30,000$$

3.2.3.6 Price and consumption

- If the price of a commodity increases by r%, then the reduction in consumption so as not to increase the expenditure, is

$$(\frac{r}{100+r} \times 100)\%$$

Example: In the new budget, the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase?

Solution: Reduction in consumption = $[\frac{R}{(100+R)}]\% = (\frac{25}{125} \times 100)\% = 20\%$

- If the price of a commodity decreases by r%, then increase in consumption, so as not to decrease expenditure on this item, is $[\frac{r}{(100-r)} \times 100]\%$

Example: In the new budget, the price of Sugar decreases by 10%. By how much percent must a person increase his consumption so that his expenditure on it does not decrease?

Solution: Increase in consumption = $\left[\frac{10}{(100-10)} \times 100\right]\%$

=11.11%

- If the price of a commodity is decreased by x % and its consumption is increased by y %,

  Or, if the price of a commodity is increased by x % and its consumption is decreased by y % then the effect on revenue

  $= \text{Inc. \% value} - \text{Dec. \% value} - \frac{\text{Inc. \% value} \times \text{Dec. \% value}}{100}$

  and the revenue is increased or decreased according to the +ve or −ve sign obtained.

Example: The tax on a commodity is diminished by 20% and its consumption increases by 15%. Find the effect on revenue.

Solution: Effect on revenue

$$= \text{Inc.\% value} - \text{Dec.\% value} - \frac{\text{Inc.\% value} \times \text{Dec.\% value}}{100}$$

$$= 15 - 20 - \frac{15 \times 20}{100} = -8\%$$

Therefore, there is a decrease of 8%.

Example: A man spends 75% of his income. His income increases by 20% and his expenditure also increases by 10%. Find the percentage increase in his savings.

Solution: Percentage increase in savings

$$= \frac{20 \times 100 - 10 \times 75}{100 - 75} = \frac{1250}{25} = 50\%$$

Example: 40 litres of a mixture of milk and water contain 10% of water. How much water must be added to make the water 20% in the new mixture?

Solution: The quantity of water to be added

$$= \frac{40(20 - 10)}{(100 - 20)} = 5 \text{litres}$$