TIME AND WORK

Topic overview – Time and Work

- ✓ **Basic Definitions**
- ✓ **Concept of unit work**
- ✓ **Two or more persons working together**
- ✓ **Variation of parameters**
- ✓ **Work efficiency**
- ✓ **Specific situations in Time and Work**
- ✓ **Illustrations**

Introduction

Tell me ! How soon can you do the job ?

How often do we hear such words at our workplace !

Mathematically, we are primarily concerned with <u>how fast</u> a piece of work can be completed with the <u>available manpower or resources</u>.
In short, we are concerned with <u>speed of doing work</u>.

Basic Definitions

- **Work** is defined as the amount of job actually done or assigned to a person or a group of persons.

➢ **Quantum of work** is usually stated in specific units suitable to the type of work or as a fraction of the total amount of work.

- **Work efficiency** is a measure of how fast a work can be done by a person or a group of persons.

➢ Thus, if A is *x* times more efficient than B, it means A will take $\frac{1}{x}^{th}$ of the time taken by B to complete the same amount of work.

Concept of unit work

- If a person can do a piece of work in ***n*** days, then the fraction of work done by him in 1 day is $\frac{1}{n}$ and the fraction of work done in ***x*** days will be $\frac{x}{n}$.

- Conversely, If a person can do $\frac{1}{n}$ part of a work in 1 day, then he can complete the work in ***n*** days.

- If ***m*** persons can do a piece of work in ***n*** days, then the fraction of work done:
  - By ***m*** persons per day = $\frac{1}{n}$
  - Per person per day = $\frac{1}{mn}$

Work done by two or more persons

- If two persons A and B can individually do a piece of work in D_1 and D_2 days respectively, then the fraction of work done by them together in 1 day is $\frac{1}{D_1}+\frac{1}{D_2}$.
- Further, A and B together will complete the work in reciprocal of $\frac{1}{D_1}+\frac{1}{D_2}$ days, that is, in $\frac{D_1D_2}{D_1+D_2}$ days.
- If A,B and C can individually do a work in D_1, D_2 and D_3 days respectively, then all of them working together can finish the work in $\frac{D_1D_2D_3}{D_1D_2+D_2D_3+D_3D_1}$ **days**

Similarly we can calculate for more than three persons.

Work done by Three persons

Example: Three workers A,B and C can complete a work individually in 3,4 and 6 days respectively. In what time will they be able to complete the work when working together ?

Solution: **A's one day work =** $\frac{1}{3}$

B's one day work = $\frac{1}{4}$

C's one day work = $\frac{1}{6}$

$\therefore$ **Combined work of A,B and C in one day =** $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{9}{12}=\frac{3}{4}$

Hence, A, B anc C can complete the work in $\frac{4}{3}$ **or** $1\frac{1}{3}$ **days.**

Two persons – Reverse situation

- If A and B can together do a piece of work in D days, and A alone can do it in D_1 days, then B alone can do the work in $\frac{DD_1}{D_1 - D}$ days

Example: **If B and C can together do a job in $2\frac{2}{5}$ days, and C alone can do the job in 4 days, then in what time will B alone be able to do the job ?**

Solution: **B alone can do the job in: $\frac{12 \times 4}{5 \times (4 - \frac{12}{5})}$ = 6 days**

Variation of parameters

Let, **W** be the amount of **work**

M be the number of **men** or workers

D be the number of **days** required

It is intuitively clear that:

More work needs more Men (& vice versa) → $W \propto M$ (D is constant)

More work needs more Time (& vice versa) → $W \propto D$ (M is constant)

→ $W \propto MD$ → $$M_1 D_1 W_2 = M_2 D_2 W_1$$

Variation of parameters

Example: 15 men can excavate a pond in 20 days. How many additional men are required to do this job in 12 days ?

Solution: Here, $M_1 = 15$, $D_1 = 20$, $D_2 = 12$, $M_2 = ?$, $W_1 = W_2 = 1$

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

$$15 \times 20 = M_2 \times 12$$

$$\therefore M_2 = 25$$

Hence, 10 more men are required to do the job in 12 days.

More parameters

Further, if we incorporate:

T : the number of working **hours per day**

E : **Work efficiency** of each worker or group of workers

The relationship becomes:

$$M_1 D_1 T_1 E_1 W_2 = M_2 D_2 T_2 E_2 W_1$$

Variation of parameters

Example: 18 men, working 4 hours a day, can do a piece of work in 15 days. Find the number of days in which 24 men, working 6 hours a day, can do twice the work. Assume that 3 men of the first group can do as much work in 2 hours, as 2 men of the second group in 5 hours.

Solution: First group: $M_1 = 18, D_1 = 15, T_1 = 4, W_1 = W_1$

Second group: $M_2 = 24, D_2 = ?, T_2 = 6, W_2 = 2\,W_1$

$E_1 : E_2 = 2 \times 5 : 3 \times 2 = 5:3$

$$M_1 D_1 T_1 E_1 W_2 = M_2 D_2 T_2 E_2 W_1$$

$18 \times 15 \times 4 \times 5 \times 2 = 24 \times D_2 \times 6 \times 3 \times 1$

$\therefore D_2 = 25$ days.

A specific situation

Example: If 15 men **or** 18 women can do a piece of work in 10 days, then how long will 15 men **and** 6 women working together take to complete the same work ?

Solution: Unitary method: Since 15 M finish the work in 10 days,

$\Rightarrow$ **15 M finish** $\rightarrow \frac{1}{10}th$ **of work per day**

$\Rightarrow$ **1 M will finish** $\rightarrow \frac{1}{10 \times 15}th = \frac{1}{150}th$ **of work per day**

Similarly, 1 W will finish $\rightarrow \frac{1}{10 \times 18}th = \frac{1}{180}$ **th of work per day**

$\Rightarrow$ **15 M + 6 W will finish** $\rightarrow \frac{15}{150} + \frac{6}{180} = \frac{2}{15}th$ **of work per day**

Hence, 15 M + 6 W will complete the work in $\frac{15}{2}$ **= 7.5 days.**

Alternate approach - Equivalence

Example: If 15 men **or** 18 women can do a piece of work in 10 days, then how long will 15 men **and** 6 women working together take to complete the same work ?

Solution: Since 15 M or 18 W finish the same work in same time,

$\therefore$ 15 M = 18 W $\Rightarrow$ 5 M = 6 W

$\Rightarrow$ 15 M + 6 W = 20 M

Since 15 M take 10 days, 20 M will take: $\frac{15 \times 10}{20}$ = 7.5 days

Hence, 15 men and 6 women can finish the work in 7.5 days.

- Equivalence approach is useful when we have different category of workers.

Direct Formula

Example: If 15 men **or** 18 women can do a piece of work in 10 days, then how long will 15 men **and** 6 women working together take to complete the same work ?

Solution: The problem may be divided into two parts:
Part-I: 1st statement containing 'or'
Part-II: 2nd statement containing 'and'

Number of days in part-II= $\dfrac{D}{\dfrac{m}{M}+\dfrac{w}{W}}$

- D → # days in part-I
- m, w → # workers in part-II
- M, W → # workers in part-I

Hence, number of days required= $\dfrac{10}{\dfrac{15}{15}+\dfrac{6}{18}} = 10\times\dfrac{3}{4} = 7.5$ **days.**

Specific situation -Working alternately

Example: Two men, A and B, working individually can mow a field in 9 and 12 days respectively. A starts mowing alone on the 1st day and thereafter they work on alternate days. In how many days will they complete mowing the field this way ?

Solution: In the first two days, fraction of work done= $\frac{1}{9}+\frac{1}{12}=\frac{7}{36}$

$\therefore$ Fraction of work done in $2 \times 5 = 10$ days $= \frac{7\times 5}{36} = \frac{35}{36}$

Balance remaining work $= 1 - \frac{35}{36} = \frac{1}{36}$

The balance work will be completed by A in $\frac{1}{36} \div \frac{1}{9} = \frac{1}{4}$ day

Hence, the work will be completed in $10\frac{1}{4}$ days.

Miscellaneous

Example: A group of men decided to do a job in 8 days. But since 10 men dropped out every day, the job got completed at the end of 12th day. How many men were there in the beginning ?

Solution: Let there be N men in the beginning.

Since, N men can do the job in 8 days,

$\therefore$ **fraction of work done per day per man =** $\frac{1}{8N}$

Hence, sum of work done for 12 days is:

$$\frac{N}{8N}+\frac{N-10}{8N}+\frac{N-20}{8N}+\ldots\ldots+\frac{N-110}{8N}=1$$

$$12N-(10+20+\ldots\ldots+110) = 8N$$

$$4N = 660 \quad \Rightarrow \quad N = 165$$

Miscellaneous

Example: One man can do as much work in one day as a woman can do in 2 days. A child does one-third the work in a day as a woman. If an estate owner hires 39 pairs of hands – men, women and children in the ratio 6:5:2 and pays them in all Rs 1,113 at the end of the day's work, what must be the daily wages of a child, if the wages are proportional to the amount of work done?

Solution: Ratio of Number of Men (M), Women (W) and Children (C)= 6:5:2

$\therefore$ **Number of M =** $\frac{6}{13} \times 39 = 18$ **Similarly, W = 15 and C = 6**

Ratio of work efficiencies, M:W:C = 6:3:1

$\therefore$Ratio of work done, M:W:C = $6\times6 : 5\times3 : 2\times1 = 36 : 15 : 2$

$\therefore$ **Proportion of wages given to Children =** $\frac{2}{53} \times 1113$ **= Rs. 42**

$\Rightarrow$ Daily wage of a child = 42 ÷ 6 = Rs 7

Thanks…