Magnetic Effect of Current

Topic covered in this lecture are:

- Ampere's Law
- Magnetic Field Inside and Outside a Long Straight Wire with Current using Ampere's Law
- Solenoids
- Toroids
- Force Between Two Parallel Currents
- Torque on a Current Loop
- Magnetic Dipole Moment
- Current-Carrying Coils
- B on Axis of Current-Carrying Coil
- Moving Coil Galvanometer

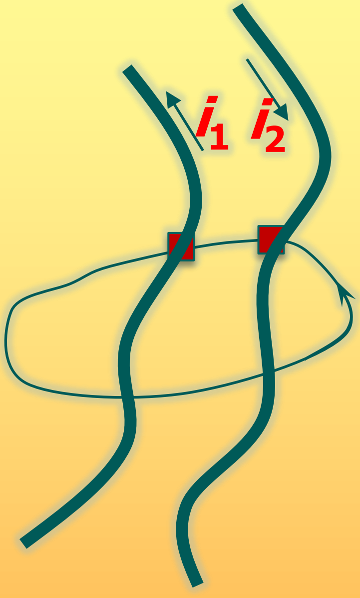
Ampere's Law

- Ampere's Law for magnetic fields is analogous to Gauss' Law for electric fields.

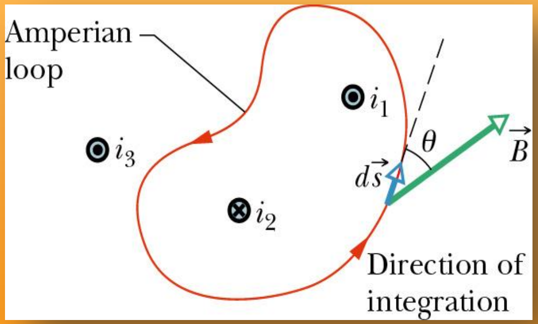
- Draw an "amperian loop" around a system of currents (like the two wires at right). The loop can be any shape, but it must be *closed*.

- Add up the component of $\vec{B}$ along the loop, for each element of length *ds* around this closed loop.

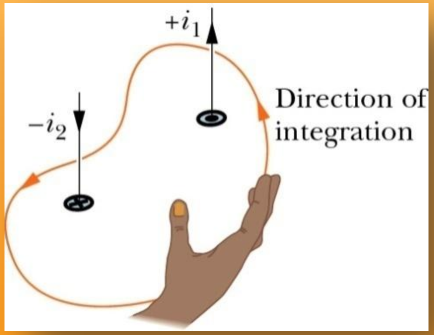
- The value of this integral is proportional to the current enclosed:



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

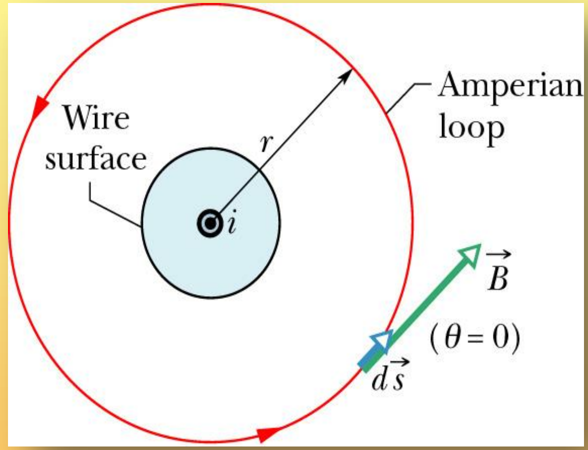


Ampere's Law



Magnetic Field Outside a Long Straight Wire with Current

- We already used the Biot-Savart Law to show that, for this case, $B = \frac{\mu_0 i}{2\pi r}$
- **Let's show it again, using Ampere's Law:**
- First, we are free to draw an Amperian loop of any shape, but since we know that the magnetic field goes in circles around a wire, let's choose a circular loop (of radius *r*).
- Then *B* and *ds* are parallel, and *B* is constant on the loop, so $\oint \vec{B} \cdot d\vec{s} = B 2\pi r = \mu_0 i_{enc}$
- And solving for B gives our earlier expression. $B = \frac{\mu_0 i}{2\pi r}$

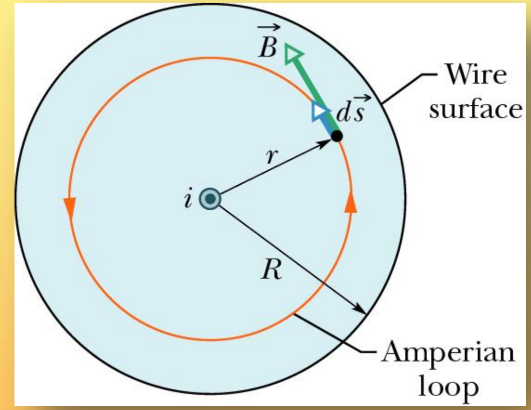


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

Ampere's Law

Magnetic Field Inside a Long Straight Wire with Current

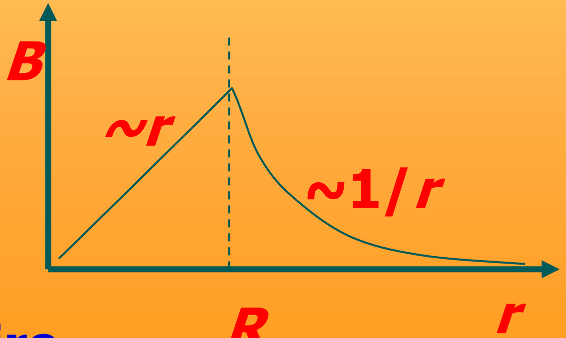
- Now we can even calculate B inside the wire.
- Because the current is evenly distributed over the cross-section of the wire, it must be cylindrically symmetric.
- So we again draw a circular Amperian loop around the axis, of radius *r* < *R*.
- The enclosed current is less than the total current, because some is outside the Amperian loop. The amount enclosed is



$$i_{enc} = i\frac{\pi r^2}{\pi R^2}$$

so $$\oint \vec{B}\cdot d\vec{s} = B2\pi r = \mu_0 i_{enc} = \mu_0 i\frac{r^2}{R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right)r$$ inside a straight wire



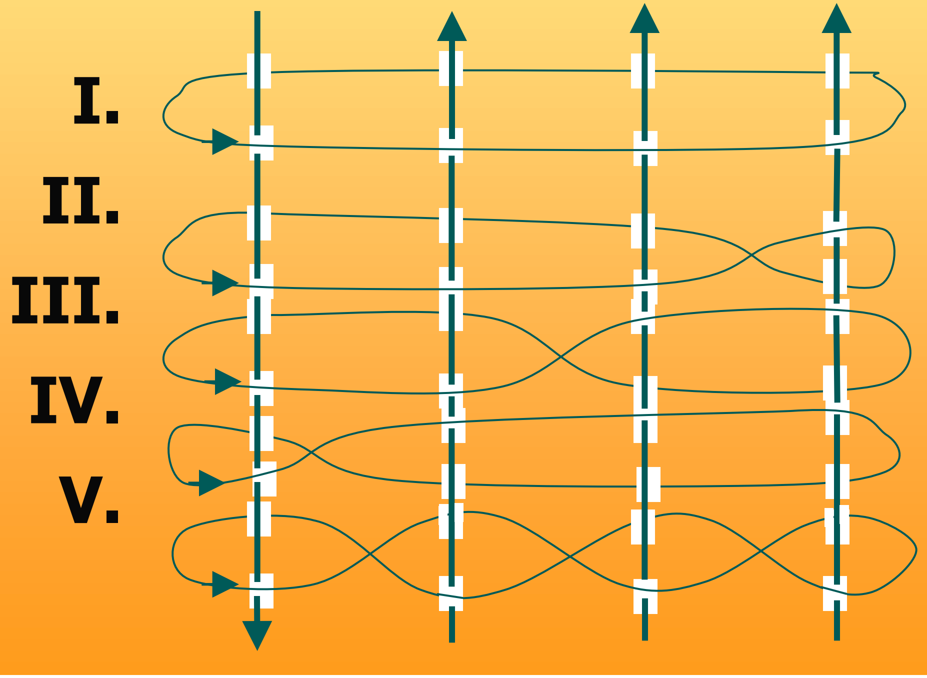
Note : Do it yourself for the outside of the wire.

Fun With Amperian Loops

Think About it!

1. **Rank the paths according to the value of $\oint \vec{B} \cdot d\vec{s}$ taken in the directions shown, most positive first.**

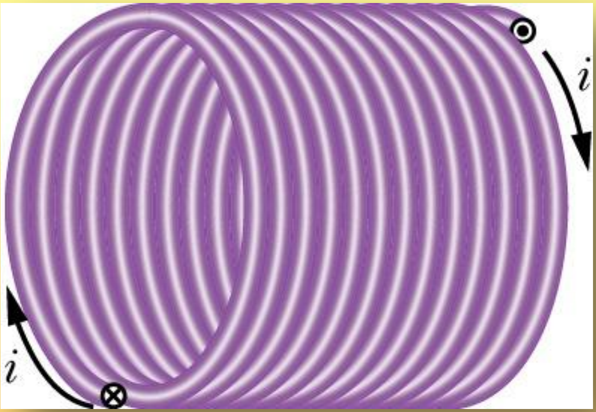
A. I, II, III, IV, V.
B. II, III, IV, I, V.
C. III, V, IV, II, I.
D. IV, V, III, II, I.
E. I, II, III, V, IV.



Solenoids

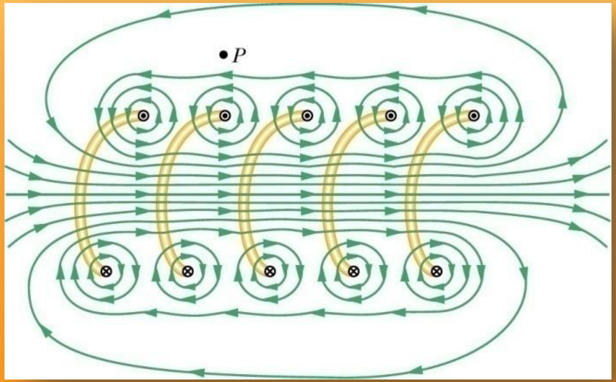
- We saw earlier that a complete loop of wire has a magnetic field at its center:

$$B = \frac{\mu_0 i}{2R}$$



- We can make the field stronger by simply adding more loops. A many turn coil of wire with current is called a solenoid.

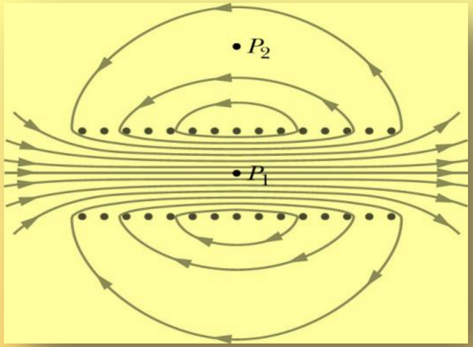
- We can use Ampere's Law to calculate B inside the solenoid.



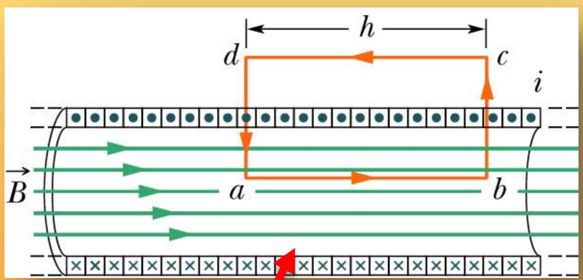
- The field near the wires is still circular, but farther away the fields blend into a nearly constant field down the axis.

Solenoids

- The actual field looks more like this:
- Compare with electric field in a capacitor.



- Like a capacitor, the field is uniform inside (except near the ends), but the direction of the field is different.
- Approximate that the field is constant inside and zero outside (just like capacitor).
- Characterize the windings in terms of number of turns per unit length, *n*. Each turn carries current *i*, so total current over length *h* is *inh*.



$$\oint \vec{B} \cdot d\vec{s} = Bh = \mu_0 i_{enc} = \mu_0 inh$$

only section that has non-zero contribution

$$B = \mu_0 in$$

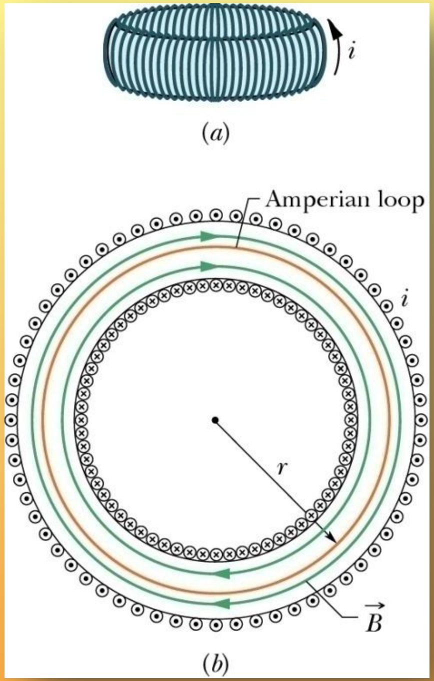
Note : At point near its end the magnetic field is given by $B = \dfrac{\mu_0 in}{2}$

Toroids

- Notice that the field of the solenoid sticks out both ends, and spreads apart (weakens) at the ends.
- We can wrap our coil around like a doughnut, so that it has no ends. This is called a toroid.
- Now the field has no ends, but wraps uniformly around in a circle.
- What is B inside? We draw an Amperian loop parallel to the field, with radius *r*. If the coil has a total of *N* turns, then the Amperian loop encloses current *Ni*.

$$\oint \vec{B} \cdot d\vec{s} = B2\pi r = \mu_0 i_{enc} = \mu_0 iN$$

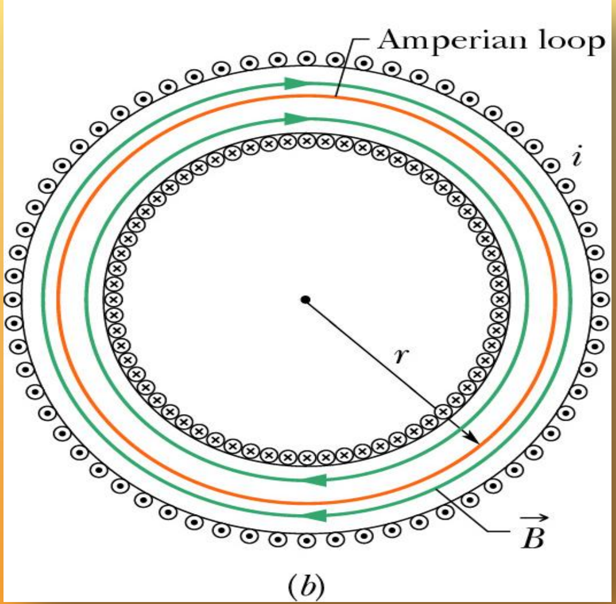
$$B = \frac{\mu_0 iN}{2\pi\, r}$$ **inside toroid**



B Outside a Toroid

Do it yourself !

2. The magnetic field inside a Toroid is $B = \dfrac{\mu_0 iN}{2\pi r}$. Using an Amperian loop, what is the expression for the magnetic field outside?



(*b*)

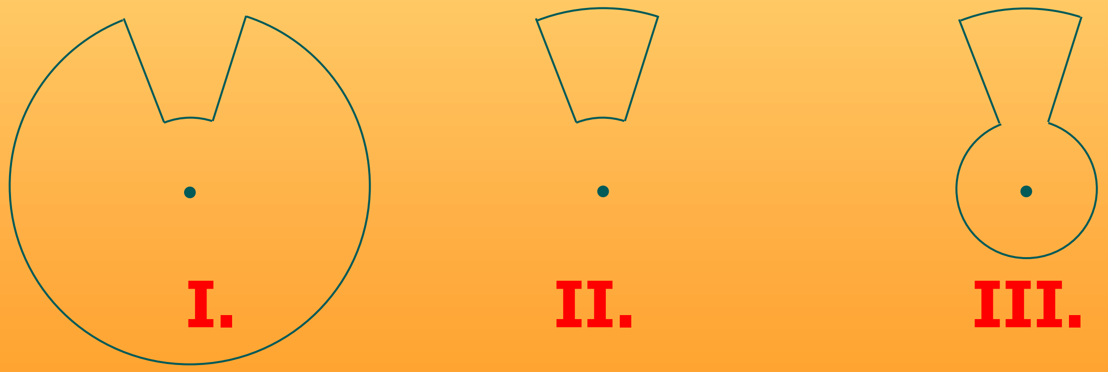
A. Zero
B. The same, decreasing as $1/r$.
C. The same, except decreasing as $1/r^2$.
D. The same, except increase as r.
E. Cannot determine.

Magnetic Field from Loops

Think about it!

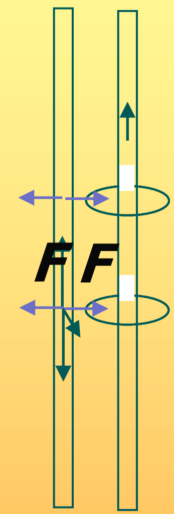
3. The three loops below have the same current. Rank them in terms of magnitude of magnetic field at the point shown, greatest first.

A. I, II, III.
B. II, I, III.
C. III, I, II.
D. III, II, I.
E. II, III, I.



Force Between Two Parallel Currents

- Recall that a wire carrying a current in a magnetic field feels a force.
- When there are two parallel wires carrying current, the magnetic field from one causes a force on the other.
- When the currents are parallel, the two wires are pulled together.
- When the currents are anti-parallel, the two wires are forced apart.



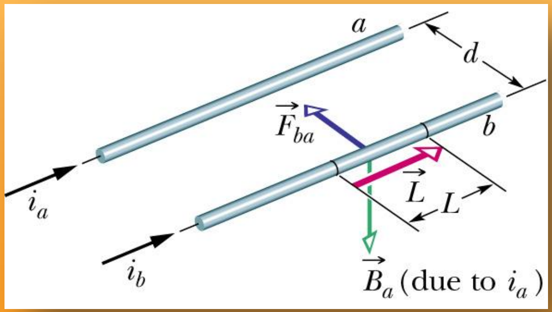
$$\vec{F}_B = i\vec{L} \times \vec{B}$$

To calculate the force on *b* due to *a*,

$$\vec{F}_{ba} = i_b\vec{L} \times \vec{B}_a$$

$$B = \frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i_a}{2\pi d}$$

$$F_{ba} = \frac{\mu_0 i_a i_b L}{2\pi d}$$

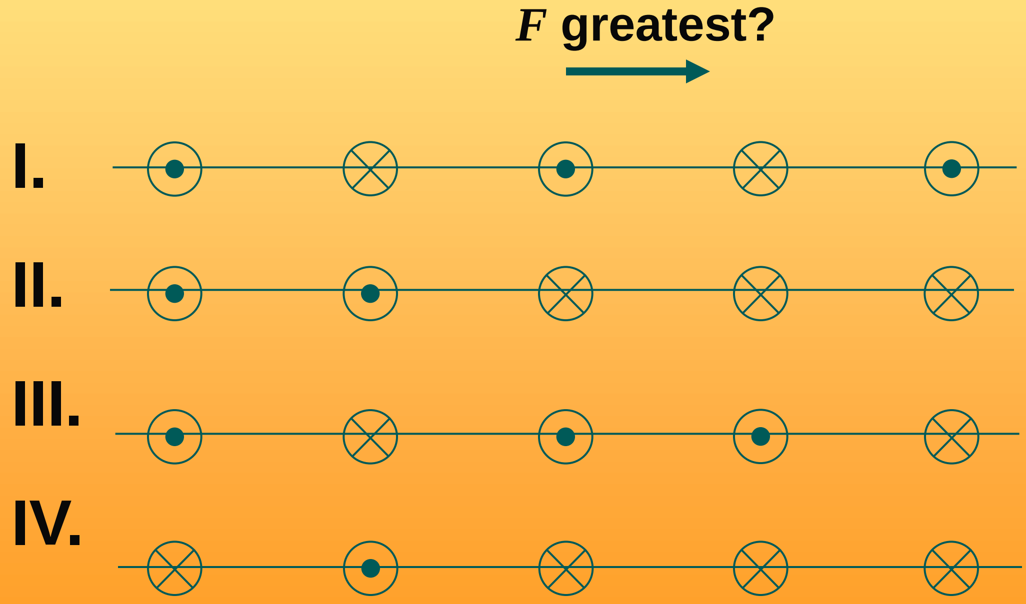


Force between two parallel currents

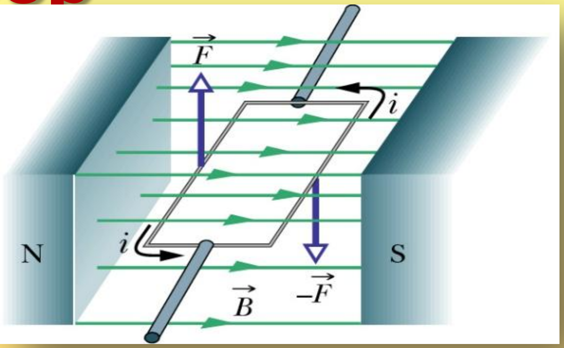
Forces on Parallel Currents

4. Which of the four situations below has the greatest force to the right on the central conductor?

A. I.
B. II.
C. III.
D. IV.
E. Cannot determine.



Torque on a Current Loop



- **Loop rotates. Calculate force for each side of the loop:**

$$F_1 = F_3 = ibB\sin(90^\circ - \theta) = ibB\sin\theta$$

$$F_2 = F_4 = iaB$$

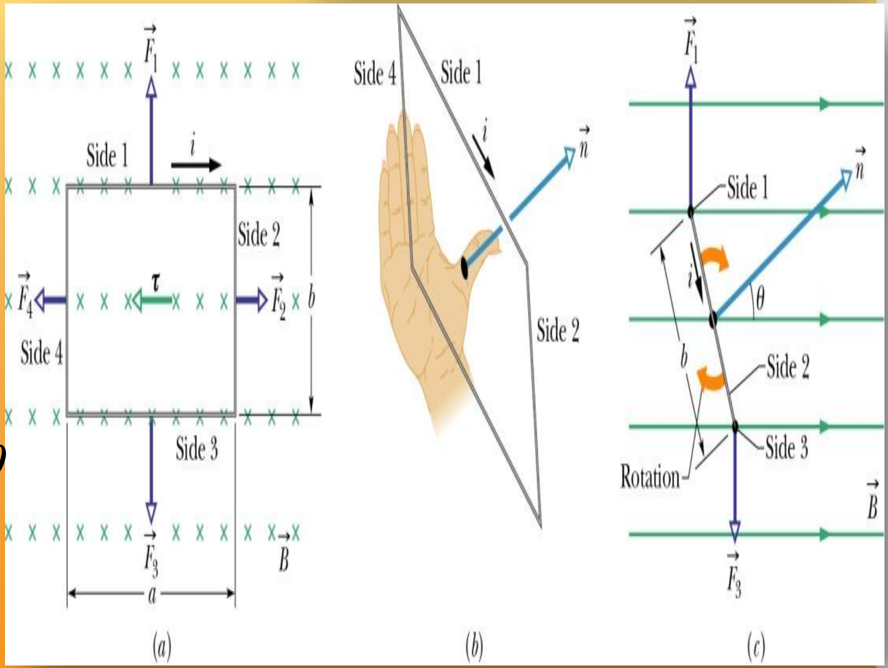
- **Torque:** $\tau = F_2 \frac{b}{2}\sin\theta + F_4 \frac{b}{2}\sin\theta$

$$= iaB\left(\frac{b}{2}\sin\theta\right) + iaB\left(\frac{b}{2}\sin\theta\right)$$

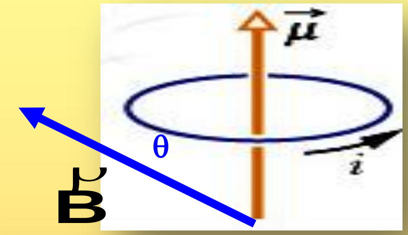
$$= iabB\sin\theta = iAB\sin\theta$$

$$\vec{\tau} = i\vec{A}\times\vec{B}$$

- **Maximum torque** $\tau_{max} = iAB$
- **Sinusoidal variation** $\tau(\theta) = \tau_{max}\sin\theta$
- **Stable when n parallels B.**
- **Restoring torque: oscillations.**

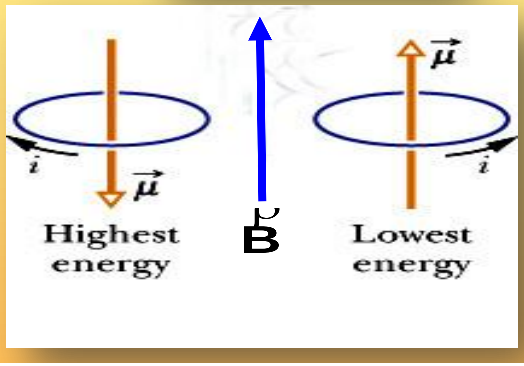


The Magnetic Dipole Moment



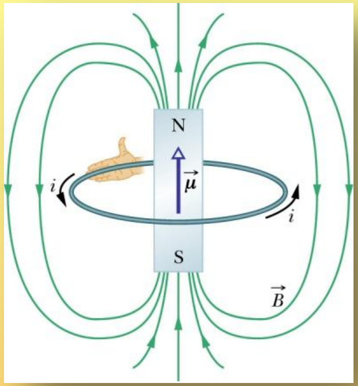
- Magnetic dipole moment $\vec{\mu} = i\vec{A}$
- SI unit: Am^2, Nm/T = J/T
- A coil of wire has N loops of the same area:
- Torque $\vec{\tau} = \vec{\mu}\times\vec{B}$ $\qquad \vec{\mu}_{coil} = Ni\vec{A}$
- Magnetic potential $U = -\vec{\mu}\times\vec{B}$
- Electric dipole and magnetic dipole

	Electric Dipole	Magnetic Dipole
Moment	$p = qd$	$\mu = NiA$
Torque	$\vec{\tau} = \vec{p}\times\vec{E}$	$\vec{\tau} = \vec{\mu}\times\vec{B}$
Potential Energy	$U = -\vec{p}\times\vec{E}$	$U = -\vec{\mu}\times\vec{B}$



Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

Current-Carrying Coils



- A current-carrying coil of wire acts like a small magnet, and we defined the "dipole moment" (a vector) as

$$|\vec{\mu}| = NiA$$

N **is number of turns,**
A **is area of loop**

- The direction is given by the right-hand rule. Let your fingers curl around the loop in the direction of *i*, and your thumb points in the direction of *B*. Notice that the field lines of the loop look just like they would if the loop were replaced by a magnet.

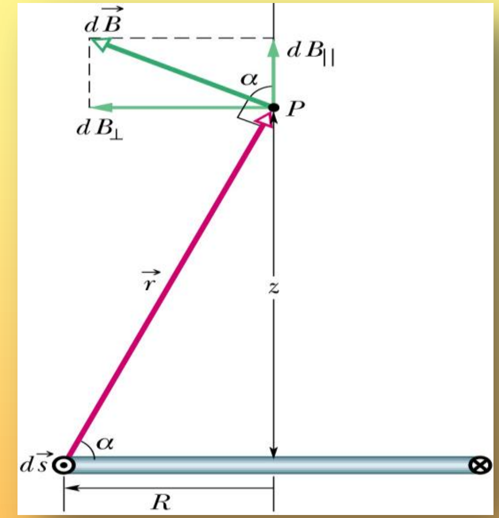
- **We are able to calculate the field in the center of such a loop, but what about other places. In general, it is hard to calculate in places where the symmetry is broken.**

- **But what about along the *z* axis?**

B on Axis of Current-Carrying Coil

- What is *B* at a point *P* on the *z* axis of the current loop?
- We use the Biot-Savart Law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}$

  to integrate around the current loop, noting that the field is perpendicular to *r*.
- By symmetry, the perpendicular part of *B* is going to cancel around the loop, and only the parallel part will survive.



$$\cos\alpha = \frac{R}{r}$$

$$dB_{||} = dB\cos\alpha = \frac{\mu_0}{4\pi}\frac{i\,ds}{r^2}\cos\alpha = \frac{\mu_0}{4\pi}\frac{i\,ds}{(R^2+z^2)^{3/2}}R$$

$$r = \sqrt{R^2+z^2}$$

$$B = \int dB_{||} = \frac{\mu_0}{4\pi}\frac{iR}{(R^2+z^2)^{3/2}}\int ds$$

$$B(0) = \frac{\mu_0 i}{2R}$$

$$B(z) = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}$$

Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer :

- **Torque experienced by the coil is**

  $\tau = N\,I\,A\,B\sin\varphi$

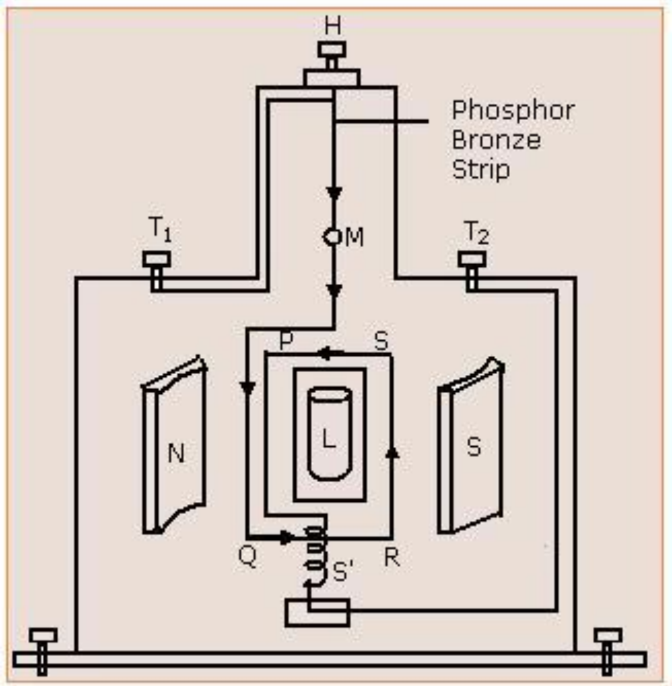
- **Restoring torque in the coil is** $\tau = k\,\alpha$ **(where k is restoring torque per unit angular twist, is the angular twist in the wire)**

- **At equilibrium,**

  $N\,I\,A\,B\sin\varphi = k\alpha$

so

$$I = \frac{K}{NAB\sin\varphi}\alpha$$

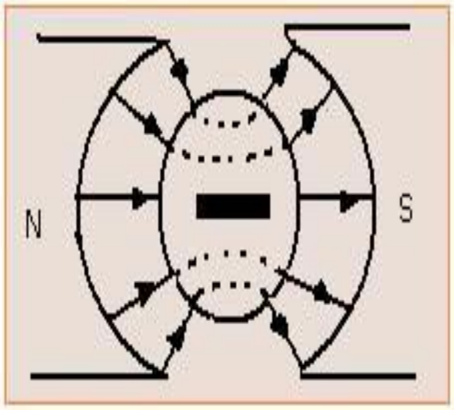


H- Torsion head, S' – Terminal screw, M- Mirror, N,S-pole -pieces of a magnet, PQRS-Rectangular coil,

Note : The factor $\sin\varphi$ can be eliminated by choos in Radial Magnetic Field.

Radial Magnetic Field:

- The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.



- So ,The angle between the plane of the coil and the magnetic field is 0°.

  or

- The angle between the normal to the plane of the coil and the magnetic field is 90°.

$$\text{i.e. } \sin\phi = \sin 90° = 1$$

$$\therefore \quad I = \frac{K}{NAB}\alpha$$

$$or \quad I = G\alpha$$

Where $G = \frac{K}{NAB}$ is called Galvanometer constant

❑ Current Sensitivity of Galvanometer:

It is the defection of galvanometer per unit current.

$$\frac{\alpha}{I} = \frac{NAB}{K}$$

❑ Voltage Sensitivity of Galvanometer:

It is the defection of galvanometer per unit voltage.

$$\frac{\alpha}{V} = \frac{NAB}{KR}$$

Example 1.: A long straight solid conductor of radius 5 cm carries a current of 2 A, which is uniformly distributed over its circular cross-section. Find the magnetic field induction at a distance of 3 cm from the axis of the conductor.

Sol. As the observation point lies inside the solid conductor, the magnetic field produced at the observation point is not due to the total current, which passes through the conductor. To find the magnetic field at a point P at distance r (= 3 cm) due to the current carrying conductor, imagine a circular path of radius r around the conductor,

such that point P lies on it. If R is the radius of the solid conductor, then current enclosed by the circular path,

$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I r^2}{R^2}$$

Let B be magnetic field at point P due to the current carrying conductor. The magnetic field B acts tangential to the circular path and its magnitude is same at every point on it.

Therefore, according to Ampere's circular law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I'$$

or

$$B \times 2 \pi r = \mu_0 \frac{I r^2}{R^2}$$

or $$B = \frac{\mu_0}{2\pi} \cdot \frac{Ir}{R^2} = \frac{\mu_0}{4\pi} \cdot \frac{2Ir}{R^2}$$

Here, I = 2 A, r = 3 cm = 0.03 m m and R = 5 cm = 0.05 m

$$\therefore \quad B = \frac{10^{-7} \times 2 \times 2 \times 0.03}{(0.05)^2} = 4.8 \times 10^{-6} T$$

Note. As the point lies inside the conductor, the magnetic field produced will be μ_r times the value obtained above, where μ_r is permeability of the material of the conductor.

Example 2.: A 0.5 m long solenoid has 500 turns and has a flux density of 2.52×10^{-3} T at its centre. Find the current in the solenoid.

Given, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}.

Sol. Here, B = 2.52 × 10-3 T ; $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Length of the solenoid, L= 0.5 m ;

total number number of turns in the solenoid, N = 500

Therefore, number of turns per unit length of the solenoid,

$$n = \frac{N}{L} = \frac{500}{0.5} = 1{,}000\ m^{-1}$$

If I is the current through the solenoid, then

$$B = \mu_0 n I$$

$$\text{or} \quad I = \frac{B}{\mu_0 n} = \frac{2.52 \times 10^{-3}}{4\pi \times 10^{-7} \times 1{,}000} = 2.0 \text{ A}$$

Example 3.: **A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3,500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid (b) inside the core of the toroid (c) in the empty space surrounded by the toroid ?**

Sol. **Here, I = 11 A; total number of turns = 3,500**
Mean radius of toroid,

$$r = \frac{25 + 26}{2} = 25.5\ \text{cm} = 25.5 \times 10^{-2}\ \text{m}$$

Total length (circumference) of the toroid

$$= 2\pi r = 2\pi \times 25.5 \times 10^{-2} = 51\pi \times 10^{-2}\ \text{m}$$

Therefore, number of turns per unit length,

$$n = \frac{3,500}{51\pi \times 10^{-2}}$$

(a) The field is non-zero only inside the core surrounded by the windings of the toroid. Therefore, the field outside the toroid is zero.

(b) The field inside the core of the toroid,

$$B = \mu_0\, n\, I = 4\pi \times 10^{-7} \times \frac{3,500}{51\pi \times 10^{-2}} \times 11 \quad = 3.02 \times 10^{-2}\ \text{T}$$

(c) For the reason given in (a), the field in the empty space surrounded by the toroid is also zero.

Example 4.: Two long straight parallel wires are 2 m apart (perpendicular to the plane of the paper) as shown in Fig 1. The wire A carries current of 9.6 A directed into the plane of the paper. The wire B carries a current, such that the magnetic field of induction at the point P at a distance 10/11 m from the wire B is zero. Find

(i) the magnitude and direction of the current in B.

(ii) the magnitude of the magnetic field of induction at the point S.

(iii) the force per unit length on the wire B.

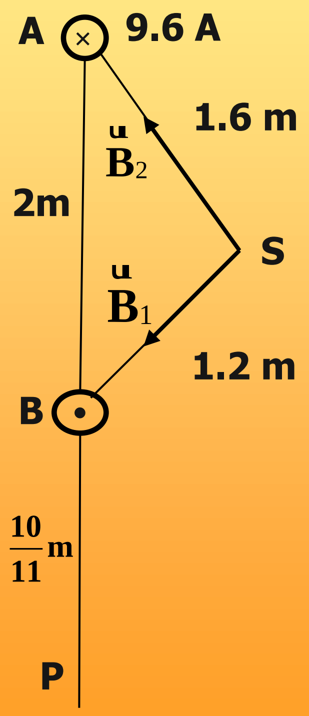


Fig. 1

Sol. Here, current through the wire A,

$I_1 = 9.6$ A (directed into the paper)

The distance of point P from the wire A,

$$a_1 = 2 + \frac{10}{11} = \frac{32}{11} \text{m}$$

and the distance of point P from the wire B,

$$a_2 = \frac{10}{11} \text{m}$$

(i) Suppose that I_2 is the current through the wire B, so that the magnetic field induction at the point P due to the two wires becomes zero. For this, current in the wire B has to be directed outwards of the paper, so that the magnetic fields produced at the point P due to the two wires are equal in magnitude and opposite in direction.

Thus, for the magnetic field induction at the point P to be zero,

$$\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{a_1} = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{a_2}$$

or $$I_2 = I_1 \times \frac{a_2}{a_1} = \frac{9.6\times 10/11}{32/11}$$

= 3 A (directed outwards of the paper)

(ii) In fact, the points A, B and S form a right angled triangle. It is because, $AB^2 = AS^2 + BS^2$.

Therefore, the magnetic field induction at the point S due to the wire A,

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{AS} = \frac{10^{-7}\times 2\times 9.6}{1.6} = 12\times 10^{-7}\,T$$

(along perpendicular to AS i.e, along SB)

The magnetic field induction at the point S due to the wire B,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{BS} = \frac{10^{-7} \times 2 \times 3}{1.2} = 5 \times 10^{-7}\,T$$

(along perpendicular to BS i.e, along SA)

As the two fields B_1 and B_2 are linclined at 90°, the resultant magnetic field at the point S is given by

$$B = \sqrt{B_1^{\,2} + B_2^{\,2}} = \sqrt{(12 \times 10^{-7})^2 + (5 \times 10^{-7})^2}$$

$$= 13 \times 10^{-7}\ T$$

(iii) The force per unit length on the wire B,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1\, I_2}{AB} = \frac{10^{-7} \times 2 \times 9.6 \times 3}{2}$$

$$= \ 2.88 \times 10^{-6}\ N\ m^{-1}$$ (*repulsive*)

Thanks…