Wave Optics

Light propagation

(1) **Different theories :**

Newtons corpuscular theory	**Huygen's wave theory**	**Maxwell's EM wave theory**	**Einstein's quantum theory**	**de-Broglie's dual theory of light**
(i) Based on Rectilinear propagation of light	(i) Light travels in a hypothetical medium ether (high elasticity very low density) as waves	(i) Light travels in the form of EM waves with speed in free space $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	(i) Light is produced, absorbed and propagated as packets of energy called photons	(i) Light propagates both as particles as well as waves
(ii) Light propagates in the form of tiny particles called Corpuscles. Colour of light is due to different size of corpuscles	(ii) He proposed that light waves are of longitudinal nature. Later on it was found that they are transverse	(ii) EM waves consists of electric and magnetic field oscillation and they do not require material medium to travel	(ii) Energy associated with each photon $E = h\nu = \frac{hc}{\lambda}$ h = planks const. $= 6.6 \times 10^{-34}\ J$ - sec ν = frequency λ = wavelength	(ii) Wave nature of light dominates when light interacts with light. The particle nature of light dominates when the light interacts with matter (micro-scopic particles)

(2) **Optical phenomena explained (√) or not explained (×) by the different theories of light**

S. No.	**Phenomena**	**Theory**				
		Corpuscular	**Wave**	**E.M. wave**	**Quantum**	**Dual**
(i)	Rectilinear Propagation	√	√	√	√	√
(ii)	Reflection	√	√	√	√	√
(iii)	Refraction	√	√	√	√	√
(iv)	Dispersion	×	√	√	×	√
(v)	Interference	×	√	√	×	√
(vi)	Diffraction	×	√	√	×	√
(vii)	Polarisation	×	√	√	×	√
(vii)	Double refraction	×	√	√	×	√
(viii)	Doppler's effect	×	√	√	×	√
(ix)	Photoelectric effect	×	×	×	√	√

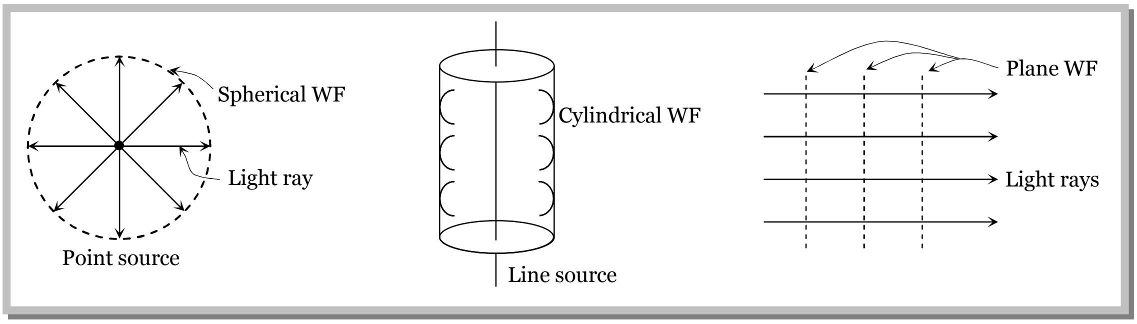
(3) **Wave front :**

(i) Suggested by Huygens

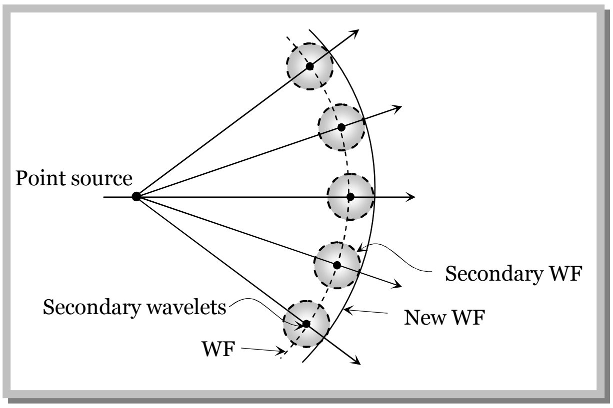
(ii) The locus of all particles in a medium, vibrating in the same phase is called Wave Front (WF)

(iii) The direction of propagation of light (ray of light) is perpendicular to the WF.

(iv)



(v) Each point on a WF acts as a source of new disturbance called Secondary wavelets. Secondary wavelets spread out as spherical secondary WF with the speed of light



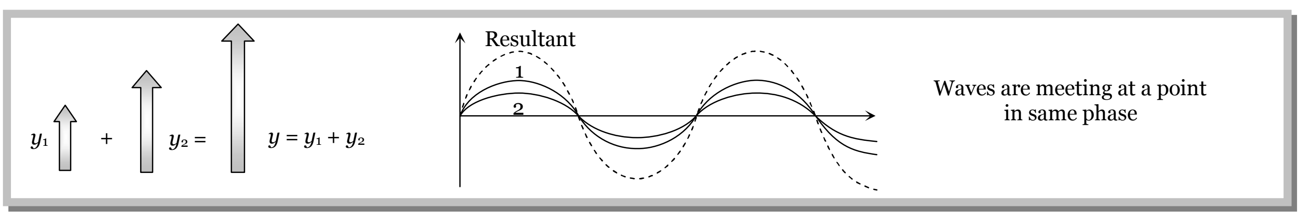
The tangential surface to all the secondary wave fronts gives new WF.

Principle of Super Position

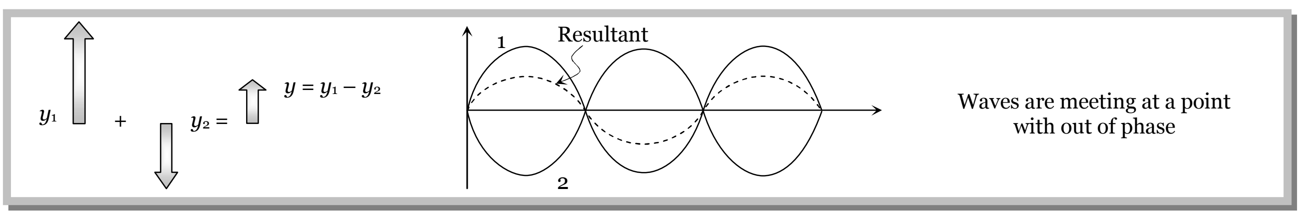
When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves. *i.e.* $\vec{y} = \vec{y}_1 + \vec{y}_2$

(1) **Graphical view :**

(i)



(ii)



(2) **Phase / Phase difference / Path difference / Time difference**

(i) Phase : The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$; term ωt = phase or instantaneous phase

(ii) Phase difference (ϕ) : The difference between the phases of two waves at a point is called phase difference *i.e.* if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference = ϕ

(iii) Path difference (Δ) : The difference in path length's of two waves meeting at a point is called path difference between the waves at that point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$

(iv) Time difference (*T.D.*) : Time difference between the waves meeting at a point is $TD = \frac{T}{2\pi} \times \phi$

(3) **Resultant amplitude and intensity :** For two superimposing waves $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ where $a_1, a_2 \rightarrow$ Individual amplitudes, $\phi \rightarrow$ Phase difference between the waves at an instant when they are meeting a point.

(i) Resultant amplitude : $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi}$ (ii) Resultant intensity : $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

Interference of Light

When two waves of exactly same frequency travels in a medium, in the same direction simultaneously then due to their superposition, at some points intensity of light is maximum while at some other points intensity is minimum. This phenomenon is called Interference of light.

(1) **Types :** It is of following two types

Constructive interference	Destructive interference
(i) When the waves meets a point with same phase, constructive interference is obtained at that point (*i.e.* maximum light)	(i) When the wave meets a point with opposite phase, destructive interference is obtained at that point (*i.e.* minimum light)
(ii) Phase difference between the waves at the point of observation $\phi = 0^o$ or $2n\pi$	(ii) $\phi = 180^o$ or $(2n-1)\pi$; $n = 1, 2, ...$ or $(2n+1)\pi$; $n = 0,1,2.....$
(iii) Path difference between the waves at the point of observation $\Delta = n\lambda$ (*i.e.* even multiple of $\lambda/2$)	(iii) $\Delta = (2n-1)\frac{\lambda}{2}$ (*i.e.* odd multiple of $\lambda/2$)
(iv) Resultant amplitude at the point of observation will be maximum $A_{max} = a_1 + a_2$ If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$	(iv) Resultant amplitude at the point of observation will be minimum $A_{min} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{min} = 0$
(v) Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{max} = 2I_0$	(v) Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$

Note :
- In interference redistribution of energy takes place in the form of maxima and minima.
- Intensity (I) $\propto$ amplitude $(a)^2$

- Average intensity : $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$
- Ratio of maximum and minimum intensities :

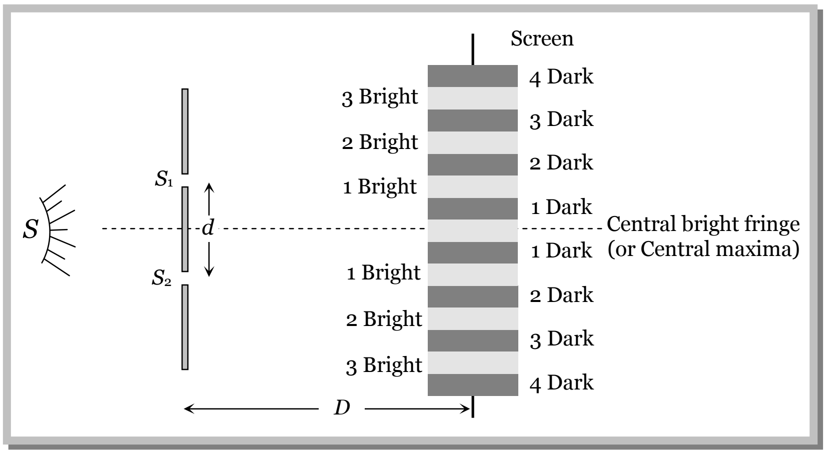
$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1}\right)^2$$

- Resultant intensity when two identical waves super imposes : If $I_1 = I_2 = I_0$ and phase difference between the waves at the point of observation is ϕ then

  Resultant intensity $I = 4I_0 \cos^2 \frac{\phi}{2}$

Young's Double Slit Experiment (YDSE)

Young's shows that when waves coming from two coherent sources (S_1, S_2) superimposes on each other, an interference pattern is obtained on the screen.



d – Distance between slits
D – Distance between slits and screen
λ – Wavelength of monochromatic light emitted from source

Note : The fringe pattern obtained due to a slit is more bright than that due to a point.

(1) **Coherent sources :** The two sources of light, whose frequencies are same and the phase difference between the waves emitted by which remains constant with time are defined as Coherent sources.

Note : Laser light is highly coherent and monochromatic.

(2) **Fringe :** In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.

(i) Central fringe is always bright, because at central position $\phi = 0^o$ or $\Delta = 0$

(ii) All fringes are of equal width. Width of each fringe is $\beta = \frac{\lambda D}{d}$ and angular fringe width $\theta = \frac{\lambda}{d} = \frac{\beta}{D}$

(iii) If the whole YDSE set up is taken in another medium then λ changes so β changes

e.g. in water $\lambda_w = \frac{\lambda_a}{\mu_w} \Rightarrow \beta_w = \frac{\beta_a}{\mu_w} = \frac{3}{4}\beta_a$

(iv) Fringe width $\beta \propto \frac{1}{d}$ *i.e.* with increase in separation between the sources, β decreases.

(v) Separation (Δx) between fringes

between n^{th} bright and m^{th} bright fringes $(n > m)$	between n^{th} bright and m^{th} dark fringe

$\Delta x = (n-m)\beta$	(a) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$ (b) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(vi) Position of any dark and bright fringe from central maxima.

(a) Position of n^{th} bright fringe from central maxima $x_n = \frac{n\lambda D}{d} = n\beta$; $n = 0, 1, 2 ....$

(b) Position of n^{th} dark fringe from central maxima $x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2}$; $n = 1, 2, 3 ....$

(3) **Condition for observing sustained interference :**

(i) The initial phase difference between the interfering waves must remain constant.

(ii) The frequency and wavelengths of two waves should be equal.

(iii) The light must be monochromatic.

(iv) The amplitudes of the waves must be equal for good contrast.

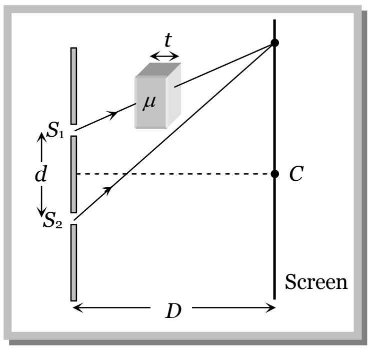
(v) The sources must be close to each other.

(vi) The sources must be narrow.

(4) **Identification of central bright fringe :** To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

(5) **Shifting of fringe pattern in YDSE :** If a transparent thin film of mica or glass is put in the path of one of the waves, then the whole fringe pattern gets shifted.

If film is put in the path of upper wave, fringe pattern shifts upward and if film is placed in the path of lower wave, pattern shift downward.



$$\text{fringe shift} = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t$$

$\Rightarrow$ Additional path difference $= (\mu - 1)t$

$\Rightarrow$ If shift is equivalent to n fringes then $n = \frac{(\mu - 1)t}{\lambda}$ or $t = \frac{n\lambda}{(\mu - 1)}$

(6) **Other important informations :**

(i) In YDSE, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_2 with light of wavelength λ_2 in the same field, then $n_1\lambda_1 = n_2\lambda_2$.

(ii) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

(iii) If slit width increases, the contrast between the fringes decreases. For very large width uniform illumination occurs.

(iv) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

(v) In YDSE if $I_1 = I_2 = I_0$ then resultant intensity at central position ($i.e.\ \phi = 0^o$) $= 4I_o$

If one of the slits is covered then screen is uniformly illuminated by light of intensity I_0. Hence intensity at central position becomes $\frac{1}{4}^{th}$.

(vii) Fringe visibility (V) : $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \sqrt{2}\,\frac{I_1 I_2}{(I_1 + I_2)}$

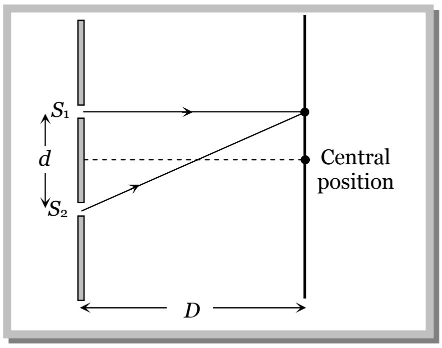
For $I_{min} = 0$, $V =$ maximum.

(viii) Missing wavelength in front of one of the slits in YDSE.

Missing wavelength at P $\boxed{\lambda = \frac{d^2}{(2n-1)D}}$

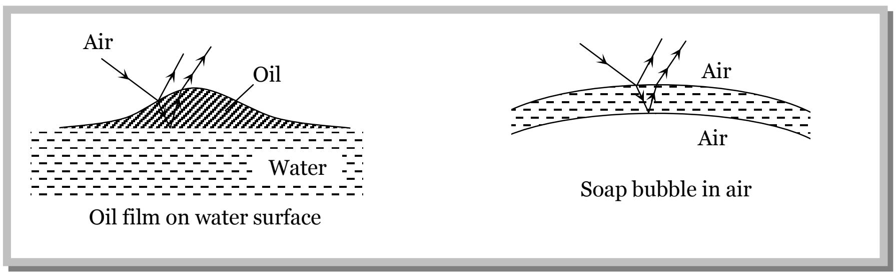
By putting $n = 1, 2, 3 \ldots.$

Missing wavelengths are $\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D} \ldots.$

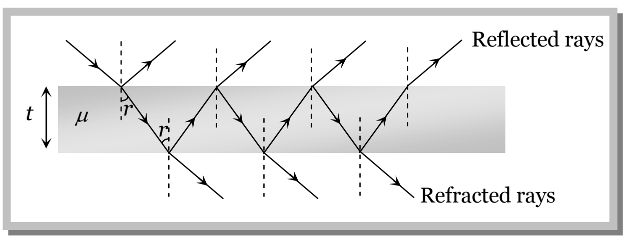


Illustrations of Interference

Interference effects are commonly observed in thin films when their thickness is comparable to wavelength of incident light (If it is too thin as compared to wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of film). Thin layer of oil on water surface and soap bubbles shows various colours in white light due to interference of waves reflected from the two surfaces of the film.



(1) **Thin films :** In thin films interference takes place between the waves reflected from it's two surfaces and waves refracted through it.

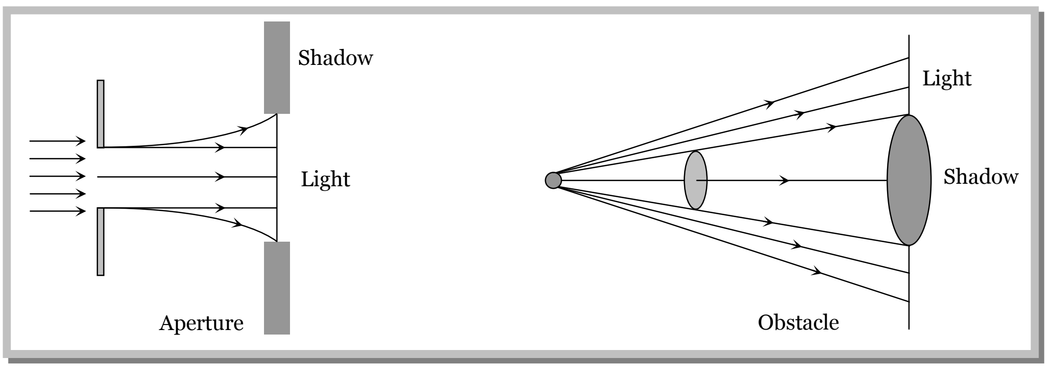


S.No.	Interference in reflected light	Interference in refracted light
(i)	Condition of constructive interference (maximum intensity) $\Delta = 2\mu\ t\cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence $r = 0$ so $2\mu t = (2n \pm 1)\frac{\lambda}{2}$	Condition of constructive interference (maximum intensity) $\Delta = 2\mu t\cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$
(ii)	Condition of destructive interference (minimum intensity) $\Delta = 2\mu t\cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$	Condition of destructive interference (minimum intensity) $\Delta = 2\mu t\cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence $2\mu t = (2n \pm 1)\frac{\lambda}{2}$

Note : ❑ The Thickness of the film for interference in visible light is of the order of $10{,}000\ \mathring{A}$.

Diffraction of Light

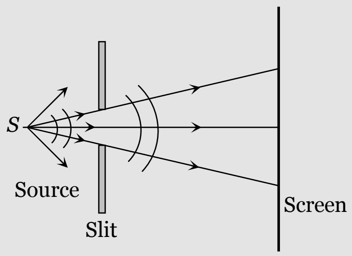
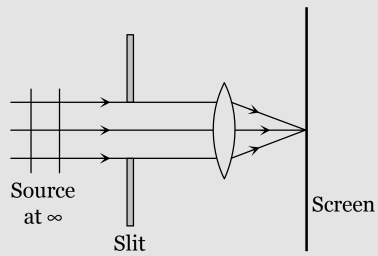
It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.



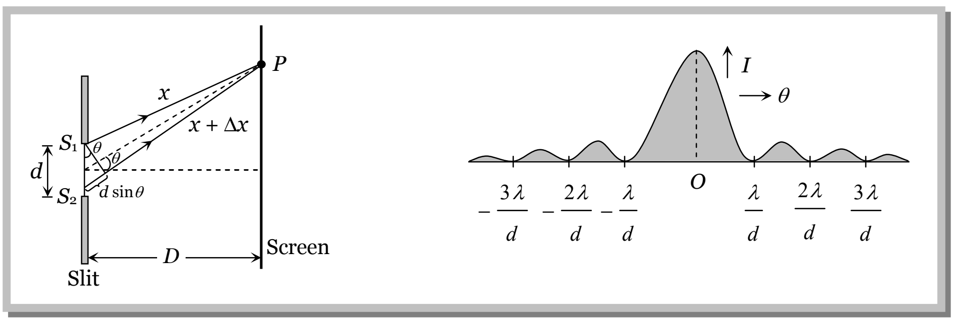
Note : ❑ Diffraction is the characteristic of all types of waves.

- ❑ Greater the wavelength of wave, higher will be it's degree of diffraction.
- ❑ Experimental study of diffraction was extended by Newton as well as Young. Most systematic study carried out by Huygens on the basis of wave theory.
- ❑ The minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size d is given by $x = \frac{d^2}{4\lambda}$.

(1) **Types of diffraction :** The diffraction phenomenon is divided into two types

Fresnel diffraction	**Fraunhofer diffraction**
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type.	(i) In this case both source and screen are effectively at infinite distance from the diffracting device.
(ii) Common examples : Diffraction at a straight edge, narrow wire or small opaque disc etc.	(ii) Common examples : Diffraction at single slit, double slit and diffraction grating.
	

(2) **Diffraction of light at a single slit :** In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown



(i) Width of central maxima $\beta_0 = \dfrac{2\lambda D}{d}$; and angular width $= \dfrac{2\lambda}{d}$

(ii) Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where $n = 1, 2, 3 ....$

i.e. $d\sin\theta = n\lambda \Rightarrow \sin\theta = \dfrac{n\lambda}{d}$

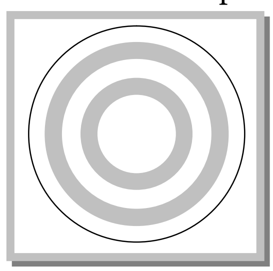
(iii) The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\dfrac{\lambda}{2}$; where $n = 1, 2, 3 ....$

i.e. $d\sin\theta = (2n+1)\dfrac{\lambda}{2} \Rightarrow \sin\theta = \dfrac{(2n+1)\lambda}{2d}$

(3) **Comparison between interference and diffraction**

S.No.	Interference	Diffraction
(i)	Results due to the superposition of waves from two coherent sources.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)
(ii)	All fringes are of same width $\beta = \frac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\frac{\lambda D}{d}$
(iii)	All fringes are of same intensity	Intensity decreases as the order of maximum increases.
(iv)	Intensity of all minimum may be zero	Intensity of minima is not zero.
(v)	Positions of nth maxima and minima $x_{n(\text{Bright})} = \frac{n\lambda D}{d}$, $x_{n(\text{Dark})} = (2n-1)\frac{\lambda D}{d}$	Positions of nth secondary maxima and minima $x_{n(\text{Bright})} = (2n+1)\frac{\lambda D}{d}$, $x_{n(\text{Dark})} = \frac{n\lambda D}{d}$
(vi)	Path difference for nth maxima $\Delta = n\lambda$	for nth secondary maxima $\Delta = (2n+1)\frac{\lambda}{2}$
(vii)	Path difference for nth minima $\Delta = (2n-1)\lambda$	Path difference for nth minima $\Delta = n\lambda$

(4) **Diffraction and optical instruments :** The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.

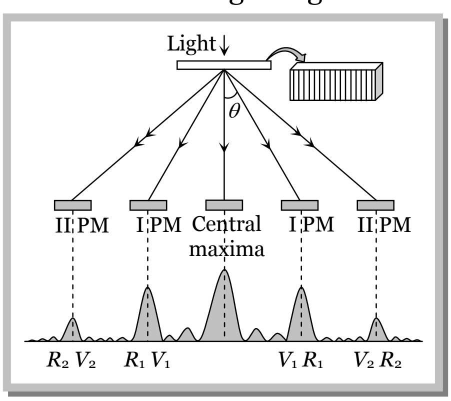


The angular half width of Airy disc $= \theta = \frac{1.22\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image $= f\theta$ (where f = focal length of the lens)

Note : ❑ Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

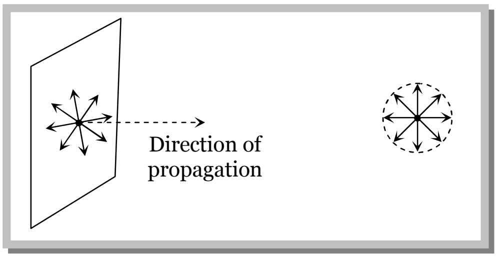
(5) **Diffraction grating :** Consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the diffraction of principle maxima (PM) is given by $d\sin\theta = n\lambda$; where d = distance between two consecutive slits and is called grating element.

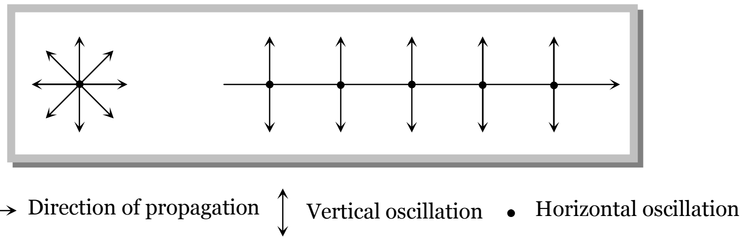


Polarisation of Light

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

(1) **Unpolarised light :** The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation is called Unpolarised light. The oscillation may be resolved into horizontal and vertical component.





(2) **Polarised light :** The light having oscillations only in one plane is called Polarised or plane polarised light.

(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

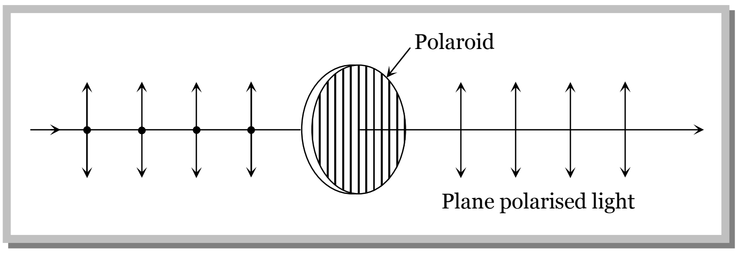
(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

(3) **Polaroids :** It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal.

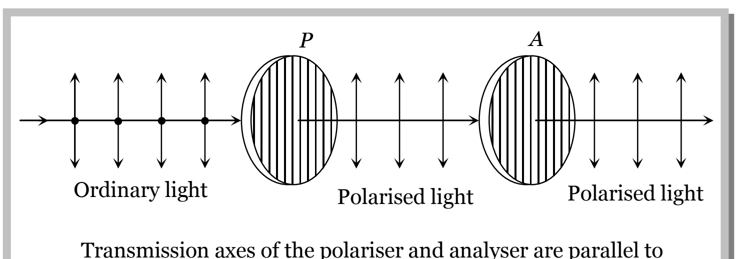
or

It is a thin film of ultramicroscopic crystals of quinine idosulphate with their optic axis parallel to each other.

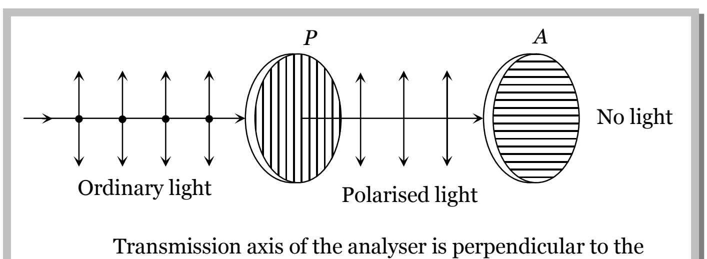


(i) Polaroids allow the light oscillations parallel to the transmission axis pass through them.

(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.



Transmission axes of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser



Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the analyser

Note : ❑ When unpolarised light is incident on the polariser, the intensity of the transmitted polarised light is half the intensity of unpolarised light.

(4) **Malus law :** This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.

(i) $I = I_0 \cos^2\theta$ and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

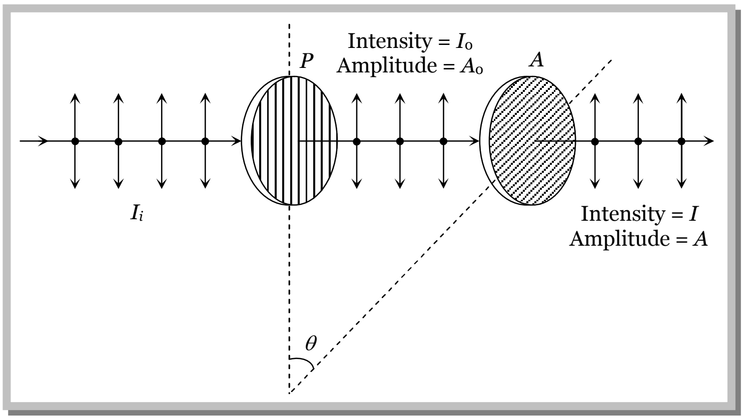
If $\theta = 0^o$, $I = I_0$, $A = A_0$

If $\theta = 45^o$, $I = \frac{I_0}{2}$, $A = \frac{A_0}{\sqrt{2}}$

If $\theta = 90^o$, $I = 0$, $A = 0$

(ii) If $I_i =$ Intensity of unpolarised light.

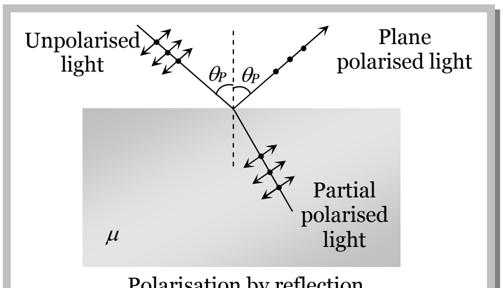
so $I_0 = \frac{I_i}{2}$ and $\boxed{I = \frac{I_i}{2}\cos^2\theta}$



Note : ❑ Percentage of polarisation $= \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$

(5) **Brewster's law :** Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index $=\mu$), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).

Also $\boxed{\mu = \tan\theta_p}$ Brewster's law



Polarisation by reflection

(6) **Use of polaroids and optical rotation :** Polaroids are used in

(i) Wind screens

(ii) Window panes of aeroplanes

(iii) Camera filters and sun glasses

(iv) 3-D movies

The phenomenon of rotation of plane of polarization by some materials while transmitting polarized light through them, is known as optical rotation and the corresponding materials are called optically active materials.

Where it is necessary to turn the analyser to the right, the substance is said be dextro-rotatory. Where it is necessary turn the analyser to the left, the substance is laevo-rotatory.

Doppler's Effect in Light

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called Doppler's effect.

If $\nu =$ actual frequency, $\nu' =$ Apparent frequency, v = speed of source *w.r.t* stationary observer, c = speed of light

Source of light moves towards the stationary observer ($v << c$)	Source of light moves away from the stationary observer ($v << c$)
(i) Apparent frequency $\nu' = \nu\left(1 - \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 - \frac{v}{c}\right)$	(i) Apparent frequency $\nu' = \nu\left(1 - \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 + \frac{v}{c}\right)$
(ii) Doppler's shift : Apparent wavelength < actual wavelength, So spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called Red shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$	(ii) Doppler's shift : Apparent wavelength > actual wavelength, So spectrum of the radiation from the source of light shifts towards the violet end of spectrum. This is called Violet shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

Note : ❑ Relation between Doppler's shift $(\Delta\lambda)$ and time period of rotation (T) of a star

$$\Delta\lambda = \frac{\lambda}{c} \times \frac{2\pi r}{T};\ r = \text{radius of star.}$$

Applications of Doppler Effect :

(i) Determination of speed of moving bodies (aeroplane, submarine etc) in RADAR and SONAR.

(ii) Determination of the velocities of stars and galaxies by spectral shift.

(iii) Determination of rotational motion of sun.

(iv) Explanation of width of spectral lines.

(v) Tracking of satellites.

(vi) In medical sciences in echo cardiogram, sonography *etc*.