CONIC SECTIONS

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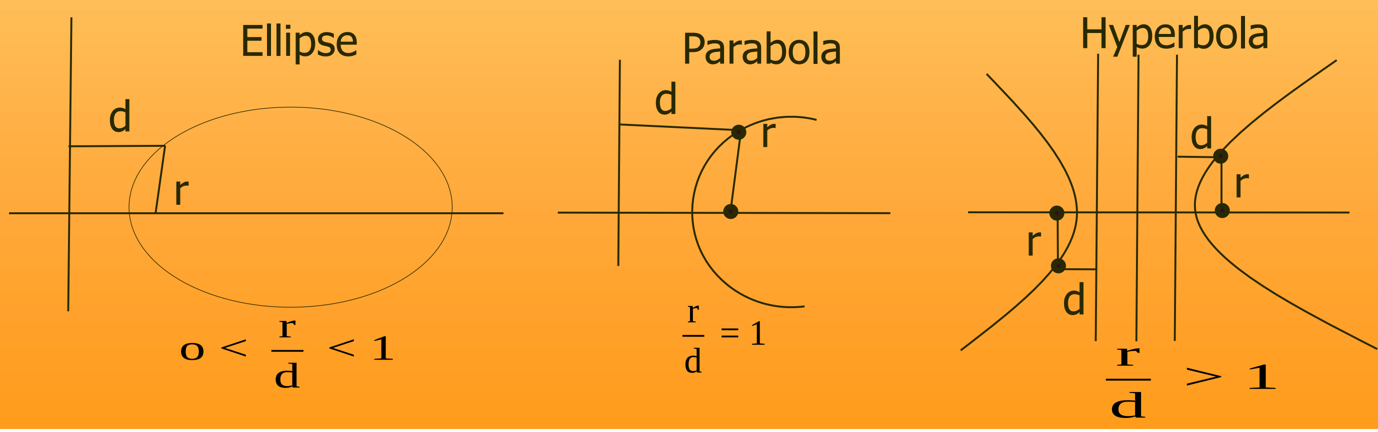
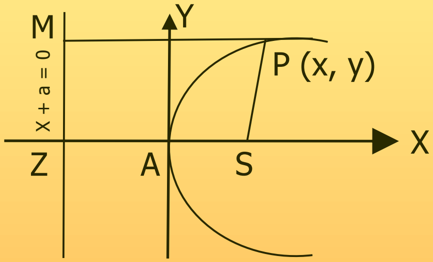
- **Conic Section** is a locus of a point which moves so that its distance from a fixed point is e times its distance from a fixed straight line.
  The fixed point is called *focus*, fixed straight line, the *directrix* and e the eccentricity of the conic. There are three cases:
  If e = 1, then conic section is known as a *parabola*.
  If e < 1, then conic section is known as an *ellipse*.
  If e > 1, then conic section is known as a *hyperbola*.

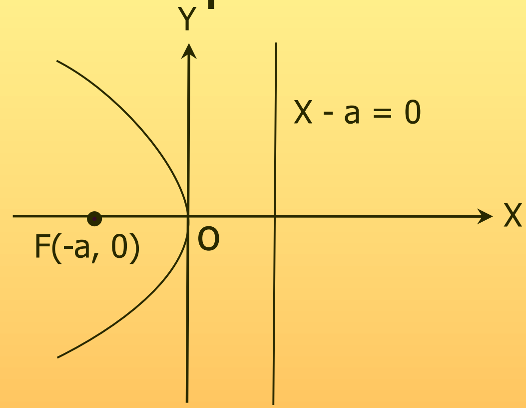


PARABOLA

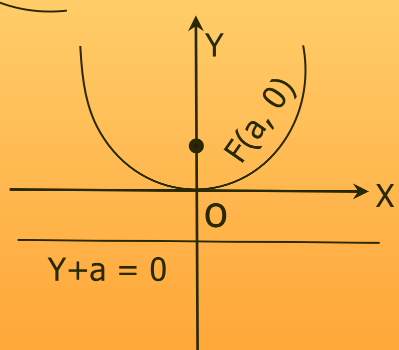
A *parabola* is the locus of a point which moves such that its distance from a fixed point is equal to its distance from a fixed straight line.



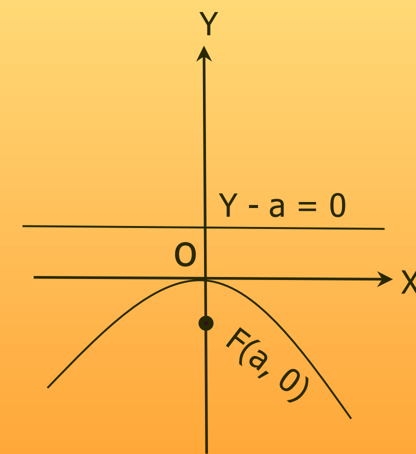
$Y^2 = 4ax$



$Y^2 = -4ax$



$X^2 = 4ay$



$X^2 = -4ay$

The fixed point is called the *focus and the* fixed straight line the *directrix.*

- **Axis:** The axis of the parabola is the straight line through the focus perpendicular to the directrix.
- **Vertex:** The vertex of a parabola is the point of intersection of the parabola with the axis.
- **Focal Distance:** The distance of the point P (x, y) of a parabola from the focus S (a, 0) is called the *focal distance* of the point P and is equal to x + a.
- **Latus Rectum:** The *latus rectum* of a parabola is the chord through the focus perpendicular to the axis.
- **Length of Latus Rectum:** The length of the latus rectum of the parabola $y^2 = 4ax$ is 4a.

- **Equation of Tangent:** Equation of the tangent at the point (x_1, y_1) on the parabola $y^2 = 4ax$ is

$$yy_1 = 2a(x + x_1)$$

- **Equation of Normal:** The equation of the normal at the point (x_1, y_1) on the parabola $y^2 = 4ax$ is

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

- **Equation of Normal in Terms of Slope:** The equation of the normal at the point $(am^2, -2am)$ on the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$
  Here $(am^2, -2am)$ are the co-ordinates of the *foot* of the normal.
  In general, three normals can be drawn from any point to the parabola.

- **Condition that a Line Touch a Parabola:** The condition that the line $y = mx + c$ may touch the parabola $y^2 = 4ax$ is $c = a/m$ and then the equation of the tangent in slope form is $y = mx + a/m$
- **Rule for a Point to Lie Outside, on or Inside a Parabola:** A point (x_1, y_1) lies outside, on or inside the parabola $y^2 = 4ax$ according as $Y_1^2 – 4ax_1 > 0, = 0 \text{ or } > 0$.
- **Parametric Representation of a Parabola:** $(at^2, 2at)$are called the parametric co-ordinates of any point on the parabola $y^2 = 4ax$.
- **Equation of Tangent in Terms of Parameter:** The equation of the tangent at the point $(at^2, 2at)$ on the parabola $y^2 = 4ax$ is $yt = x + at^2$.

- **Equation of Chord in Terms of Midpoint:** The equation of the chord of the parabola $y^2 = 4ax$, whose mid-point is (x_1, y_1) is

  $$yy_1 - 2ax = Y_1^2 - 2ax_1 \text{ or } T = S_1.$$

- **Equation of Chord of Contact:** The equation of the chord of contact of tangents from (x_1, y_1) to the parabola $y^2 = 4ax$ is

  $$yy_1 = 2a(x + x_1) \text{ or } T = 0$$

- **Polar of a Point with Respect to a Parabola:** The equation of the polar of the point (x_1, y_1) with respect to the parabola $y^2 = 4ax$ is

  $$yy_1 = 2a(x + x_1)$$

ELLIPSE

An *ellipse* is the locus of a point which moves so that its distance from a fixed point is in a constant ratio, less than one, to its distance from a fixed straight line.

- ✓ The fixed point is called the *focus*.
- ✓ The constant ratio is called the eccentricity and is denoted by e.
- ✓ The fixed straight line is called the *directrix*.

- **Equation of Ellipse:** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

  Vertices (± a, 0) , Foci (± ae, 0)
  and Directrices $x=\pm\frac{a}{e}$.

(i) The length of the latus rectum is $\frac{2b^2}{a}$.

(ii) The focal distance of any point P on the ellipse is the distance of the point P from the focus, i.e., PF = 2a.

- **Parametric Coordinates:** Let P be a point on the ellipse with eccentric angle θ. Then (a cos θ, b sin θ) are called *parametric co-ordinates* of the point P. In short, this point is called the point θ.

- **Equation of Tangent and Normal**

✓ in *Cartesian coordinates* at any point (x_1, y_1):

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1, \frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

✓ In *Parametric coordinates* at any point 'θ':

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1, \text{ax sec}\theta-\text{by cosec}\theta=a^2-b^2$$

- **Equation of the Chord of the Ellipse joining the Points θ_1 and θ_2:** The equation of the chord joining the points whose coordinates are (a cos $\theta_{1,}$ b sin θ_1) and (a cos $\theta_{2,}$ b sin θ_2) on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } \frac{x}{a}\cos\frac{\theta_1+\theta_2}{2}+\frac{y}{b}\sin\frac{\theta_1+\theta_2}{2}=\cos\frac{\theta_1-\theta_2}{2}$$

- **Equation of Chord of Ellipse with given Point as its Middle –point:** The equation of a chord of the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose mid-point is ($x_1$ y_1), is

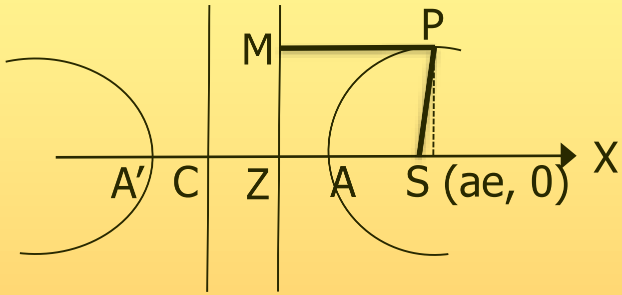
$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=\frac{x_1^{\ 2}}{a^2}-\frac{y_1^{\ 2}}{b^2} \text{ or } T = S_1$$

- **Equation of Chord of Contact:** The equation of the chord of contact of tangents from the point (x_1, y_1), to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1 \text{ or } T = 0$$

HYPERBOLA

A hyperbola is the locus of a point which moves so that its distance from a fixed point is in a constant ratio, greater than one, to its distance from a fixed straight line.



- ✓ The fixed point is called the *focus*, and is denoted by S.
- ✓ The constant ratio is called the *eccenricity* and is denoted by e.
- ✓ The fixed straight line is called the *directrix*.

Standard equation of Hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- ✓ A, A′ are called vertices of the hyperbola, whose co-ordinates are (a, 0) and (-a, 0) respectively.
- ✓ AA′ is called the *transverse axis*.
- ✓ CY is called the *conjugate axis*.
- ✓ C is called the centre of the *hyperbola*.

- **Length of Latus Rectum:** The length of the latus rectum of the hyperbola is $\frac{2b^2}{a}$.
- **Focal Distance of any Point on a Hyperbola:** The focal distance of any point P (x, y) on the hyperbola is ex $\pm$ a.
- **Equation of Tangent:** The equation of tangent at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

- **Equation of Normal:** The equation of the normal at the point (x_1, y_1) of the hyperbola is $\frac{x - x_1}{x_1 / a^2} = \frac{y - y_1}{y_1 / b^2}$
- **Condition that the line** y = mx + c **may touch the hyperbola**

$$c = \pm\sqrt{a^2m^2 + b^2}$$

- **Point (x_1, y_1) lies outside, on, or inside the hyperbola** according as $\frac{x - x_1}{x_1 / a^2} = \frac{y - y_1}{y_1 / b^2}$ is positive, zero or negative.

- **Director Circle:** The locus of the point of intersection of two perpendicular lines to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 - b^2$.

- **Parametric Coordinates:** The parametric co-ordinates of any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are (a sec θ, b tan θ).

- **Equation of the chord joining two points** 'θ'$_1$ and 'θ'$_2$

$$\frac{x}{a}\cos\frac{\theta_1+\theta_2}{2} - \frac{y}{b}\sin\frac{\theta_1+\theta_2}{2} = \cos\frac{\theta_1+\theta_2}{2}$$

- **Equation of the normal at any point 'θ' on the hyperbola** ax cos θ + bycot θ = a_2 + b_2.

- **Equation of chord of contact of tangent from the point (x_1, y_1) to the hyperbola** $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

Equation of asymptotes to the hyperbola: $y = \pm \frac{b}{a}$

- **Auxiliary Circle:** A circle described on the transverse axis AA′ of a hyperbola as a diameter is called auxiliary circle.
- **Rectangular Hyperbola:** A hyperbola whose asymptotes are at right angles to each other is called a rectangular hyperbola.
- **Equation of a Rectangular Hyperbola** $x^2 - y^2 = a^2$.
- **Eccentricity of a Rectangular Hyperbola referred to asymptotes as axes $xy = c^2$, $e = \sqrt{2}$**

- **Equation of Tangent:** Equation of tangent at the point (x_1, y_1) on the rectangular hyperbola $xy = c^2$ is $\frac{x}{x_1} + \frac{y}{y_1} = 2$

- **Equation of Normal:** Equation of normal at the point (x_1, y_1) on the rectangular hyperbola $xy = c^2$ is $xx_1 - yy_1 = x_1^2 - y_1^2$.

- **Parametric Co-ordinates:** The parametric co-ordinates of any point on a rectangular hyperbola $xy = c^2$ are defined as $(ct, c/t)$.

- **Equatio of Tangent at any Point 't':** The equation of tangent at any point $(ct, c/t)$ on a rectangular hyperbola $xy = c^2$ is

$$x + yt^2 - 2ct = 0.$$

- **Equation of Normal:** The equation of normal at any point $(ct, c/t)$ on a rectangular hyperbola $xy = c^2$ is

$$xt^3 - yt - c\,(t^4 - 1) = 0.$$

Thank You…