Dynamics

Definitions.

(1) **Speed :** The speed of a moving point or particle is the rate at which the point of the particle is describing its path or speed is the distance covered by a body in one second. It is a scalar quantity.

(2) **Displacement :** The displacement of a moving point is its change of position of a body. It is a vector quantity.

(3) **Velocity :** It is the rate of change of displacement of a body. It is a vector quantity.

(4) **Acceleration :** The acceleration of a moving point, particle or a body is the rate of change of its velocity. It is a vector quantity. Acceleration $f = \frac{v_2 - v_1}{t}$.

(5) **Retardation :** Decreasing, the acceleration is said to be negative. The negative acceleration is some times called a retardation.

(6) **Uniformly accelerated motion :** A point or a particle moves in a straight line, starting with initial velocity *u*, and moving with constant acceleration *f* in its direction of motion, If *v* be its final velocity at the end of time *t*, and *s* be its distance at the instant from its starting point, then

$v = u + ft$ …..(i) $s = ut + \frac{1}{2}ft^2$ …..(ii), $v^2 = u^2 + 2fs.$ …..(iii)

(7) **Distance travelled in any particular second :** Distance travelled by the body or moving particle in t^{th} second is given by, $s_t = u + \frac{1}{2}f(2t-1).$

Motion under gravity.

When a body is let fall in vacuum towards the earth, it will move vertically downward with an acceleration which is always the same at the same place on the earth but which varies slightly form place to place. This acceleration is called acceleration due to gravity. Its value in M.K.S. system is $9.8\,m/\sec^2$, in C.G.S. system $981\,cm/\sec^2$ and in F.P.S. system $32\,ft/\sec^2$. It is always denoted by *g*.

(1) **Downward motion**: If a body is projected vertically downward from a point at a height *h* above the earth's surface with velocity *u*, and after *t* second its velocity becomes *v*, the equation of its motion are $v = u + gt$,

$h = ut + \frac{1}{2}gt^2$, $v^2 = u^2 + 2gh$, $s_t = u + \frac{1}{2}g(2t-1)$. In particular, if the body starts from rest or is simply let fall or dropped, then, $v = gt$, $h = \frac{1}{2}gt^2$, $v^2 = 2gh$. $(\because u = 0)$

(2) **Upward motion :** When a body be projected vertically upward from a point on the earth's surface with an initial acceleration due to gravity and if the direction of the upward motion is regarded as + ve, the direction of acceleration is –*ve* and it is, therefore, denoted by *g*. The body thus moves with a retardation and its velocity gradually becomes lesser and lesser till it is zero. Thus, for upward motion, the equations of motion are,

$$v = u - gt, \quad h = ut - \frac{1}{2}gt^2, \quad v^2 = u^2 - 2gh, \quad s_t = u - \frac{1}{2}g(2t-1).$$

(3) **Important deductions** : (i) **Greatest height attained** : Let *H* be the greatest height. From the result, $v^2 = u^2 - 2gh$. We have, $0 = u^2 - 2gH \quad \therefore \quad H = \frac{u^2}{2g}$.

Hence greatest height, $(H) = \frac{u^2}{2g}$.

(ii) **Time to reach the greatest height :** Let *T* be the time by the particle to reach the greatest height.

From the result, $v = u - gt$

We have, $0 = u - gT$ i.e., $T = \frac{u}{g}$. Therefore time to reach the greatest height $(T) = \frac{u}{g}$

(iii) **Time of flight :** It is the total time taken by the particle to reach the greatest height and then return to the starting point again. When the particle returns to the starting point, $h = 0$. Therefore, from the result, $h = ut - \frac{1}{2}gt^2$.

We have, $0 = ut - \frac{1}{2}gt^2$ or $t=0$ or $\frac{2u}{g}$

$t = 0$ corresponds to the instant with the body starts, and $t = 2\,u/g$ corresponds to the time when the particle after attaining the greatest height reaches the starting point. $\therefore$ time of flight $= \frac{2u}{g}$

Laws of motion.

(1) **Newton's laws of motion :** (i) **First law :** Everybody continues in its state of rest or to uniform motion in a straight line except in so far as it to be compelled by external impressed forces to change that state.

(ii) **Second law :** The rate of change of momentum is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts.

$$\frac{d}{dt}(mv) \propto F \Rightarrow m\frac{dv}{dt} \propto F \quad \Rightarrow \quad m\frac{dv}{dt} = KF \Rightarrow F = mf$$

Force = mass $\times$ acceleration

The unit of force are Dyne or Newton or Poundals.

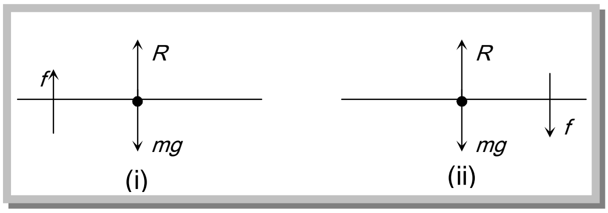
(iii) **Third law :** To every action there is an equal and opposite reaction, or the actions and reaction are always equal and opposite.

(2) **Momentum of a body :** It is the quantity of motion in a body and is equal to the product of its mass (*m*) and velocity (*v*) with which it moves. Thus, momentum of the body is *mv*. The unit of momentum are *gm-cm/sec* or *kg-m/sec*.

(3) **Apparent weight in a lift when in motion :** Let a body of mass m be placed in a lift moving with an acceleration *f*, and *R* is the normal reaction, then

(i) For an upward acceleration: $R = m(g+f)$.

(ii) For downward acceleration: $R = m(g-f)$.



Projectile motion

When a body is thrown in the air, not vertically upwards but making an acute angle α with the horizontal, then it describes a curved path and this path is a Parabola.

The body so projected is called a projectile. The curved path described by the body is called its trajectory.

The path of a projectile is a parabola whose equation is $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$

Its vertex is $A\left(\frac{u^2\sin\alpha\cos\alpha}{g}, \frac{u^2\sin^2\alpha}{2g}\right)$

Focus is $S\left(\frac{u^2\sin 2\alpha}{2g}, \frac{-u^2\cos 2\alpha}{2g}\right)$

Some important deductions about projectile motion :

(1) Time of flight $= \dfrac{2u\sin\alpha}{g}$

(2) Range of flight *i.e.*, Horizontal Range $R = \dfrac{u^2 \sin 2\alpha}{g}$

Maximum horizontal Range $= \dfrac{u^2}{g}$ and this happens when $\alpha = \dfrac{\pi}{4}$

(3) Greatest height $= \dfrac{u^2 \sin^2 \alpha}{2g}$

Time taken to reach the greatest height $= \dfrac{u\sin\alpha}{g}$

(4) Let $P(x, y)$ be the position of the particle at time *t*. Then $x = (u\cos\alpha)t$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

(5) Horizontal velocity at time *t*$= \dfrac{dx}{dt} = u\cos\alpha$

Vertical velocity at time *t*$= \dfrac{dy}{dt} = u\sin\alpha - gt$

(6) Resultant velocity at time *t*$= \sqrt{u^2 - 2ugt\sin\alpha + g^2t^2}$

And its direction is $\theta = \tan^{-1}\left(\dfrac{u\sin\alpha - gt}{u\cos\alpha}\right)$

(7) Velocity of the projectile at the height *h* $= \sqrt{u^2 - 2gh}$

and its direction is $\theta = \tan^{-1}\left[\dfrac{\sqrt{u^2\sin^2\alpha - 2gh}}{u\cos\alpha}\right]$

(8) Range and time of flight on an inclined plane with angle of inclination β are given by

$$R = \frac{2u^2\cos\alpha\sin(\alpha - \beta)}{g\cos^2\beta}$$

and maximum range up the plane $= \dfrac{u^2}{g(1+\sin\beta)}$, where $2\alpha - \beta = \dfrac{\pi}{2}$

Time of flight $T = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$

Range down the plane $= \dfrac{2u^2\cos\alpha\sin(\alpha + \beta)}{g\cos^2\beta}$

Maximum range down the plane $= \dfrac{u^2}{g(1-\sin\beta)}$, where $2\alpha + \beta = \dfrac{\pi}{2}$

Time of flight $= \dfrac{2u\sin(\alpha+\beta)}{g\cos\beta}$

(9) Condition that the particle may strike the plane at right angles is $\cot\beta = 2\tan(\alpha-\beta)$

Collision of elastic bodies.

(1) **Elasticity :** It is that property of bodies by virtue of which they can be compressed and after compression they recover or tend to recover their original shape. Bodies processing this property are called elastic bodies.

(2) **Law of conservation of momentum** : It states "the total momentum of two bodies remains constant after their collision of any other mutual action." Mathematically $m_1.u_1 + m_2.u_2 = m_1.v_1 + m_2.v_2$

where $m_1 =$ mass of the first body

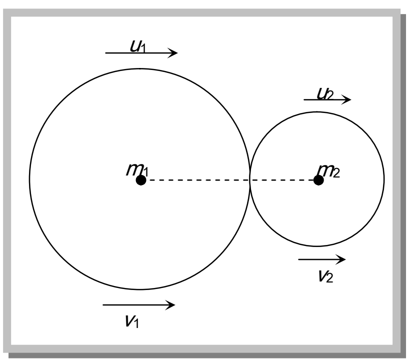
$u_1 =$ initial velocity of the first body

$v_1 =$ final velocity of the first body.

$m_2, u_2, v_2 =$ Corresponding values for the second body.

(3) **Coefficient of restitution :** This constant ratio is denoted by *e* and is called the coefficient of restitution or coefficient of elasticity. The values of e varies between the limits 0 and 1.

(4) **Direct impact of two smooth spheres :** Two smooth spheres of masses m_1 and m_2 moving along their line centres with velocities u_1 and u_2 (measured in the same sense) impinge directly. To find their velocities immediately after impact. *e* being the coefficient of restitution between them.



Let v_1 and v_2 be the velocities of the two spheres immediately after impact measured along their line of centres in the same direction in which u_1 and u_2 are measured. As the spheres are smooth, the impulsive action and reaction between them will be along the common normal at the point of contact. From the principle of conservation of momentum,

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \qquad \text{.....(i)}$$

Also from Newton's experimental law of impact of two bodies,

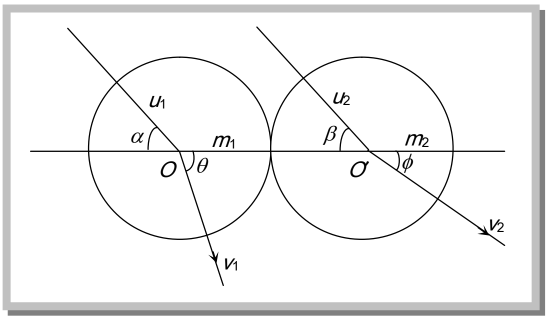
$$v_2 - v_1 = e[u_1 - u_2] \qquad .....(ii)$$

Multiplying (ii) by m_2 and subtracting from (i), we get $\Rightarrow (m_1 + m_2)v_1 = (m_1 - em_2)u_1 + m_2 u_2 (1+e)$

$$\therefore \quad v_1 = \frac{(m_1 - em_2)u_1 + m_2 u_2 (1+e)}{m_1 + m_2} \qquad .....(iii)$$

(5) **Oblique impact of two spheres :** A smooth spheres of mass m_1, impinges with a velocity u_1 obliquely on a smooth sphere of mass m_2 moving with a velocity u_2. If the direction of motion before impact make angles α and β respectively with the line joining the centres of the spheres, and if the coefficient of restitution be e, to find the velocity and directions of motion after impact,

Let the velocities of the sphere after impact be v_1 and v_2 in directions inclined at angles θ and ϕ respectively to the line of centres. Since the spheres are smooth, there is no force perpendicular to the line joining the centres of the two balls and therefore, velocities in that direction are unaltered.



$$v_1 \sin\theta = u_1 \sin\alpha \qquad .....(i)$$

$$v_2 \sin\phi = u_2 \sin\beta \qquad .....(ii)$$

And by Newton's law, along the line of centres,

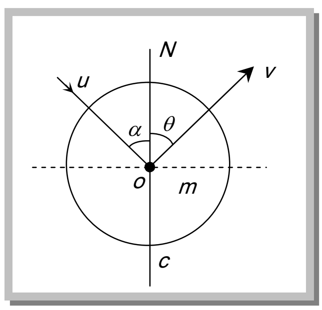
$$v_2 \cos\phi - v_1 \cos\theta = -e(u_2 \cos\beta - u_1 \cos\alpha) \qquad .....(iii)$$

Again, the only force acting on the spheres during the impact is below along the line of centres. Hence the total momentum in that in unaltered.

$$\therefore m_1 v_1 \cos\theta + m_2 v_2 \cos\phi = m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta \qquad .....(iv)$$

The equations (i), (ii), (iii) and (iv) determine the unknown quantities.

(6) **Impact of a smooth sphere on a fixed smooth plane :** Let a smooth sphere of mass *m* moving with velocity *u* in a direction making an angle α with the vertical strike a fixed smooth horizontal plane and let *v* be the velocity of the sphere at an angle θ to the vertical after impact.



Since, both the sphere and the plane are smooth, so there is no change in velocity parallel to the horizontakl plane. $\therefore v\sin\theta = u\sin\alpha$ .....(i)

And by Newton's law, along the normal *CN*, velocity of separation = *e*.(velocity of approach)

$$\therefore v\cos\theta - 0 = eu\cos\alpha$$

$$v\cos\theta = eu\cos\alpha \qquad \text{....(ii)}$$

Dividing (i) and (ii), we get $\cot\theta = e\cot\alpha$

Particular case :– If $\alpha = 0$ then from (i) $v\sin\theta = 0 \Rightarrow \sin\theta = 0$

$$\therefore \theta = 0$$

$\because v \neq 0$ and from (ii) $v = eu$

Thus if a smooth sphere strike a smooth horizontal plane normally then it will rebound along the normal with velocity, *e* times the velocity of impact.

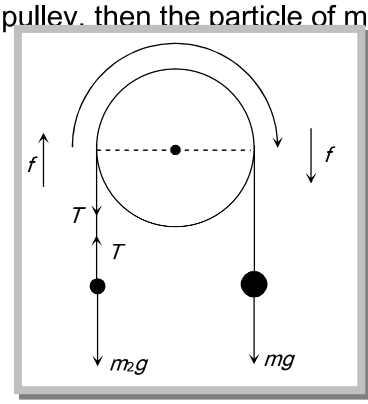
i.e. velocity of rebound = *e*.(velocity before impact)

Rebounds of a particle on a smooth plane :– If a smooth ball falls from a height *h* upon a fixed smooth horizontal plane, and if *e* is the coefficient of restitution, then whole time before the rebounding ends $= \sqrt{\left(\frac{2h}{g}\right)}.\frac{1+e}{1-e}$

And the total distance described before finishing rebounding $= \frac{1+e^2}{1-e^2}h$

Motion of two particles connected by a string.

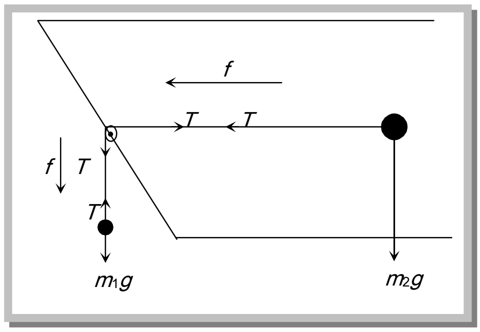
(1) **Two particles hanging vertically :** If two particles of masses m_1 and m_2 $(m_1 > m_2)$ are suspended freely by a light inextensible string which passes over a smooth fixed light pulley, then the particle of mass $m_1(> m_2)$ will move downwards, with Acceleration $f = \frac{(m_1 - m_2)g}{m_1 + m_2}$



Tension in the string $T = \frac{2m_1 m_2 g}{m_1 + m_2}$

Pressure on the pulley $= 2T = \frac{4m_1 m_2}{m_1 + m_2}.g$

(2) **One particle on smooth horizontal table :** A particle of mass m_1 attached at one end of light inextensible string is hanging vertically, the string passes over a pulley at the end of a smooth horizontal table, and a particle of mass m_2 placed on the table is attached at the other end of the string. Then for the system,

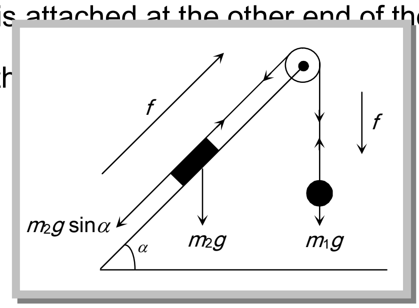


Acceleration $f = \frac{m_1 g}{m_1 + m_2}$

Tension in the string $T = \frac{m_1 m_2 g}{m_1 + m_2}$

Pressure on the pulley $= T\sqrt{2} = \frac{\sqrt{2} m_1 m_2 g}{m_1 + m_2}$

(3) **One particle on an inclined plane :** A particle of mass m_1 attached at one end of a light inextensible string is hanging vertically, the string passes over a pulley at the end of a smooth plane inclined at angle α to the horizontal, and a particle of mass m_2 placed on this inclined plane is attached at the other end of the string. If the particle of mass m_1 moves vertically downwards, then for th



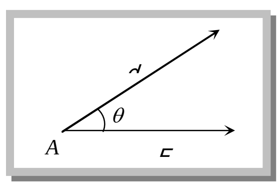
Acceleration $f = \frac{(m_1 - m_2 \sin\alpha)}{m_1 + m_2} g$

Tension in the string $T = \frac{m_1 m_2 (1 + \sin\alpha)}{m_1 + m_2} g$

Pressure on the pulley $= \sqrt{\{2(1 + \sin\alpha)\}} T = \frac{\sqrt{2} m_1 m_2 g (1 + \sin\alpha)^{3/2}}{m_1 + m_2}$

Work, Power, Energy

(1) **Work :** Work is said to be done by a force when its point of application undergoes a displacement. In other words, when a body is displaced due to the action of a force, then the force is said to do work. Work is a scalar quantity.



Work done = force $\times$ displacement of body in the direction of force

$W = \vec{F}.\vec{d}$

$W = F d \cos\theta$

where θ is the angle between *F* and *d*.

(2) **Power :** The rate of doing work is called power. It is the amount of work that an agent is capable of doing in a unit turn. 1 watts = 10^7 ergs per *sec* = 1 *joule* per *sec*., 1 H.P. = 746 watts.

(3) **Energy :** Energy of a body is its capacity for doing work and is of two kinds-

(i) **Kinetic energy :** is the capacity for doing work which a moving body possesses by virtue of its motion. And is measured by the work which the body can do against any force applied to stop it, before the velocity is destroyed.

$$K.E. = 1/2mv^2$$

(ii) **Potential energy :** The potential energy of a body is the capacity for doing work which it possesses by virtue of its position of configuration. $P.E. = mgh$.