Properties of Liquids

Contents

2. PROPERTIES OF LIQUIDS

2.1 FLUID STATICS

2.1.1 Fluid

A fluid is a substance, such as liquid or gas that has no rigidity like solids. Liquids are distinguished from gases by the presence of a surface. As fluids have no rigidity, they fail to support a shear stress. When a fluid is subjected to a shear stress, the layers of the fluid slide relative to each other. This characteristic gives the fluid, an ability to flow or change shapes.

Note: Fluids obey Newton's laws of motion. Any infinitesimal volume of fluid accelerates according to net force acting on it.

Under static condition, the only force component that needs to be considered is one that acts normal or perpendicular to the surface of the fluid. No matter what the shape of a fluid, forces between the interior & exterior are at right angles to the fluid boundary.

The magnitude of normal force per unit surface area is called pressure. Pressure is a scalar quantity and has no directional properties.

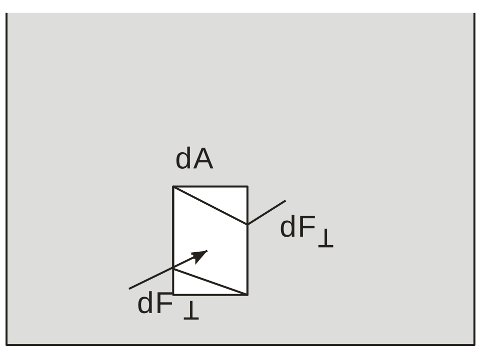
The force $\overrightarrow{\Delta F}$ exerted by a fluid against the fluid surface depends on the pressure P according to

$$\overrightarrow{\Delta F} = P\overrightarrow{\Delta A}.$$

Where $\overrightarrow{\Delta A}$ is the area vector (taken along the outward normal).

2.1.2 Fluid Pressure

Consider an elemental area dA inside a fluid, the fluid on one side of area dA presses the fluid on the other side and vice-versa. We define the pressure p a point as the normal force per unit area, i.e.



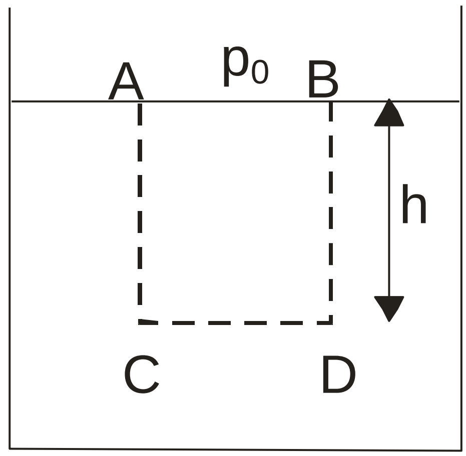
$p = dF_{\perp}/dA$

If the pressure is same at all the points of a finite plane surface with area A, then

$p = F_{\perp}/dA$; where $F_{\perp}$ is the normal force on one side of the surface.

The SI unit of pressure is 'pascal'; where 1 pascal = 1 Pa = 1 N/m^2

The pressure inside a fluid can be calculated by considering the following.



If we analyze the force acting on surface CD we have to take into account the weight of the fluid column just above CD (other parts of the liquid cannot exert any force because liquid cannot exert shear stress).

Considering the pressure at the free surface to be p_0, the force acting on the layer of fluid at CD

$$= p_0A + \rho = ghA = (p_0 + \rho gh) A$$

where ρ = density of liquid.

Hence, pressure acting at any point on CD $= p_0 + \rho gh$.

Question: Does a fluid exert force on the surface of a container in which it is kept?

Answer: Fluids exert force on all surfaces in contact with them. They exert thrust in a direction normal to the surface in contact. To understand this; let us consider a surface in contact with a fluid. Let A be the area of the surface in contact. If the fluid pressure P on this area is same everywhere, then thrust on the surface F=PA along the normal to the surface.

Thrust due to fluid = ρghA = Vρg,

where V = Volume of the fluid.

Hence, the thrust on a surface depends on the volume of fluid above it. This suggests that if a body is immersed in a fluid the thrust on the lower surface will be more than that on the upper surface.

2.1.3 Atmospheric Pressure

Atmospheric pressure is defined as the force per unit area exerted against a surface by the weight of air above that surface at any given point in the Earth's atmosphere. Normal atmospheric pressure at sea level (an average value) is 1 atmosphere (atm) which is equal to 1.013×10^5 Pa.

- Fluid force acts perpendicular to any surface in the fluid, not matter how that surface is oriented. Hence, pressure has no intrinsic direction of its own, i.e. it is a scalar.
- The excess pressure above atmospheric pressure is called gauge pressure, and total pressure is called absolute pressure

2.1.4 Variation of pressure with depth

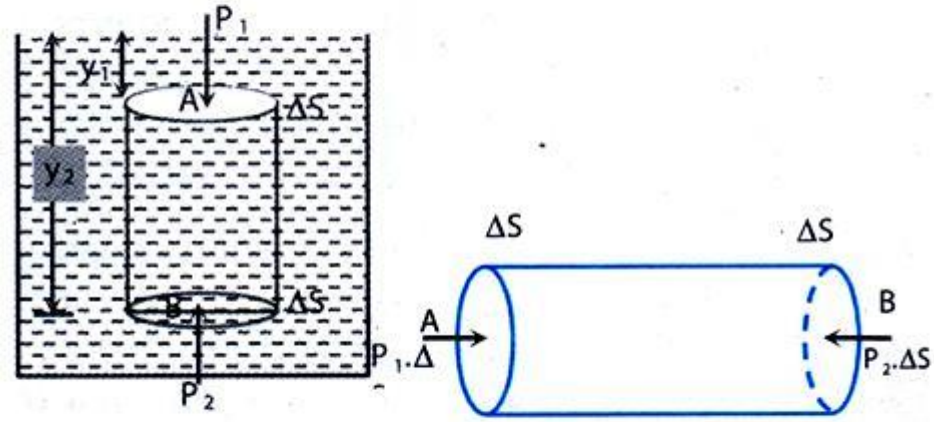
(i) Let pressure at A is P_1 and pressure at B is P_2

Then, $P_2 \Delta S = P_1 \Delta S + \rho g \Delta S (y_2 - y_1)$

Or $P_2 = P_1 + \rho g (y_2 - y_1)$

The pressure increases with depth (y),

i.e. $dP/dy = \rho g$; where ρ = density of the fluid (kg/m^3).



(ii) Pressure is same at two points in the same horizontal level.

As body is in equilibrium, $P_1 \Delta S = P_2 \Delta S$

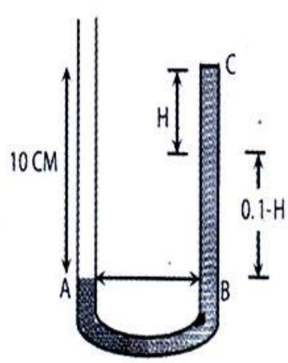
or $P_1 = P_2$

Example: A vertical U-tube of uniform cross-section contains mercury in both arms. A glycerin (relative density = 1.30 column of length 10 cm is introduced into one of the arms. Oil of density 800 kg m^{-3} is poured into the other arm until the upper surface of the oil and glycerin are the same horizontal level. Find the length of the oil column. Density of mercury is 13.6×10^3 kgm^{-3}.

Solution: Pressure at A and B must be same.

Pressure at A = $p_0 + 0.1 \times (1.3 \times 1000) \times g$

Where p_0 = atmospheric pressure



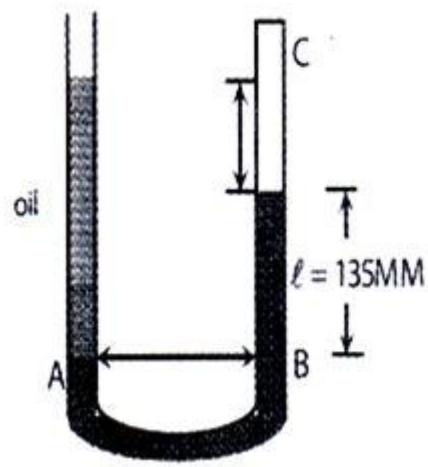
Pressure at B = $p_0 + h \times 800 \times g + (0.1 - h) \times 13.6 \times 1000\ g$

$= p_0 + 0.1 \times 1300 \times g$

$= p_o + 800\ gh + 1360\ g - 13600 \times g \times h$

or $h = 9.6$ cm

Example: For the arrangement shown in the figure, what is the density of oil?



Solution: $p_{surface} = p_o + \rho_w \cdot gl$

$p_{surface} = p_o + \rho_{oil}\ (l + d)g$

$\Rightarrow \rho_{oil} = (\rho_\omega . l)/(l+d)$

$= 1000(135)/(135+12.3) = 916\ kg/m^3$

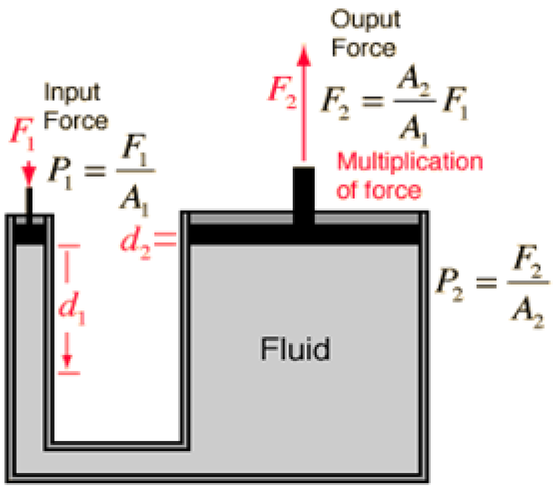
2.2 PASCAL'S LAW

Pascal's Law states that if you apply pressure to fluids that are confined (or can't flow to anywhere), the fluids will then transmit (or send out) that same pressure in all directions at the same rate.

Have you ever stepped on a balloon? Remember how the balloon bulged out on all sides under your foot - not just on one side? That is Pascal's Law in action! The air was confined by the balloon, and you applied pressure with your foot.

2.2.1 Applications of Pascal's law

1. Hydraulic lift



$$F_1 d_1 = F_2 d_2$$

$$d_1 = \frac{F_2}{F_1} d_2 = \frac{A_2}{A_1} d_2$$

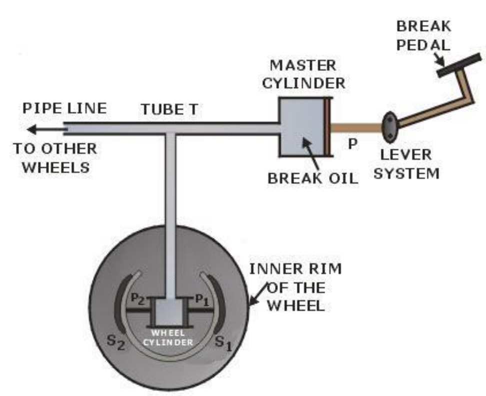
A multiplication of force can be achieved by the application of fluid pressure according to Pascal's principle, which for the two pistons implies

$$P_1 = P_2$$

This allows the lifting of a heavy load with a small force, but of course there can be no multiplication of work, so in an ideal case with no frictional loss:

$$W_{input} = W_{output}$$

2. Hydraulic Break



Hydraulic Brake

The wheel cylinder of hydraulic drum brakes acts as a double hydraulic press, multiplying the force on the fluid by the ratio of the area of the cylinder to the area of the supply line.

Besides the multiplication of force achieved, Pascal's principle guarantees that the pressure is transmitted equally to all parts of the enclosed fluid system. This gives straight-line braking unless there is a fluid leak or something to cause a significant difference in the friction of the surfaces.

2.3 ARCHIMEDES'S PRINCIPLE

Archimedes's principle states that the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.

When a body is immersed partially or wholly in to a liquid, it experiences an upward thrust, which is equal to the weight of liquid displaced by the body.

Here the up thrust is called buoyant force. It acts through the centre of buoyancy which is centre of gravity of the displaced liquid.

Apparent weight = True weight - Up thrust

If a body of volume V and density d_1 is fully immersed in a liquid of density d_2, then

True weight $W = Vd_1g$

Weight of displaced liquid $= Vd_2g$

Apparent weight $W_1 = Vd_1g - Vd_2g$

$W_1 = V d_1 g (1-d_2/d_1)$; $\Rightarrow W_1 = W (1-d_2/d_1)$

a) If $d_2 < d_1$, $W_1 > 0$ and body will sink to the bottom.
b) If $d_2 = d_1$, $W_1 = 0$ and the body will just float or remain hanging at whatever position it is left inside the liquid.
c) If $d_2 > d_1$, $W_1 < 0$ i.e. up thrust will be greater than the true weight. The body will move partly out of the free surface of the liquid until the up thrust becomes equal to true weight W.

2.3.1 Laws of flotation

a) Weight of the floating body is equal to weight of liquid displaced.
b) Centre of gravity of the floating body and centre of gravity of the displaced liquid are in same vertical line.

If a body of volume V_1 and density d_1 floats in a liquid of density d_2 and V_2 is the volume of the body immersed in the liquid, then

$$V_1 d_1 g = V_2 d_2 g$$

$$\Rightarrow \quad V_1 d_1 = V_2 d_2$$

If the liquid is water, relative density or specific gravity $= V_2 / V_1$

$$\text{Specific gravity of a liquid} = \frac{\text{Loss of weight in liquid}}{\text{loss of weight in water}}$$

When a body floats to 1/x part of its volume in a liquid of density d_1 and to 1/ y part in a liquid of density d_2, then

$$\frac{x}{y} = \frac{d_1}{d_2}$$

If W is weight of a body in air, W_1 is its weight in a liquid and W_2 is weight of that body in water (in both cases completely immersed)

$$Relative\ density\ of\ liquid = \frac{W - W_1}{W - W_2}$$

A beaker contains a liquid. An ice block is floating on the surface of that liquid. When ice melts completely, (d_l, d_w are densities of liquid and water respectively)

a) Level of the liquid rises if $d_l > d_w$

b) Level of the liquid falls if $d_l < d_w$

c) Level of the liquid does not change if $d_l = d_w$

A block of ice with steel balls inside floats on water, the level falls when ice melts. If the ice block has an air bubble or cork, the level does not change.

If a body of outer volume V has a cavity of volume V_0 in it,

$$V = \frac{W_{air} - W_{water}}{d_w g}$$

$$V - V_0 = \frac{m}{d} = \frac{mass\ of\ body\ in\ air}{density\ of\ material}$$

A block is placed in a vessel in contact with the base. When liquid is poured into that vessel, so that liquid does not go beneath it, no buoyant force acts on it. The downward force on it is (P_0+ hdg) A (on upper surface, where d is density of liquid)

Example: Consider a container floating on water. Now we slowly pour water into it. The container goes down. Why?

Solution: Initially, force of buoyancy = weight of container.

Later, force of buoyancy = weight of container + weight of water in it.

Hence, to increase the force of buoyancy the depth upto, which the cylinder is, submerged increase, as this will displace more water.

Apparent decrease in weight of the body = upthrust = weight of liquid displaced by the body

Example: A pipe of copper having an internal cavity weighs 264 gm in air and 221 gm in water. Find the volume of the cavity. [Density of copper is 8.8 gm/cc.]

Solution: The buoyant force on the copper piece, F = Vσg

Hence, volume of the copper piece V = F/σg = (264-221) g/1xg = 43 cc

The volume of the material of the copper piece

V_0 = mass of copper piece / density of material = 264/8.8 = 30 cc

Hence, volume of the cavity = V - V_0 = 43 - 30 = 13 cc

2.4 VISCOSITY

Viscosity is an internal property of a fluid that offers resistance to flow.

2.4.1 Coefficient of viscosity

The ratio of shearing stress to the shear rate is called coefficient of viscosity.

Shearing stress = F/A

$$\text{strain rate} = \frac{\text{change in shear strain}}{\text{time interval}} = \frac{\Delta x/l}{\Delta t} = \frac{v}{l}$$

$$\therefore \text{Coefficient of viscosity}, \eta = \frac{F/A}{v/l} = \frac{Fl}{vA}$$

Here v/l is the velocity gradient between two layers of liquid

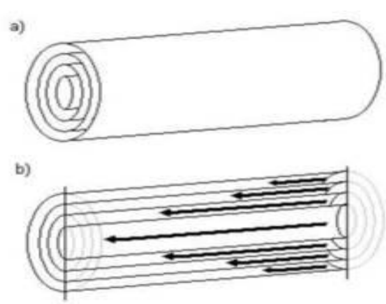
Unit: poiseuille (Pl) or Pa-s or N $m^{-2}s$ or kg m^{-1} s^{-1}

Dimensional formula: $[ML^{-1}T^{-1}]$

2.4.2 Similarity between viscosity and solid friction

1. Both comes into play when there is a relative motion
2. Both oppose relative motion
3. Both are due to molecular forces
4. Both act tangentially in a direction opposite to that of motion

2.4.3 Poiseuille's formula



Poiseuille studied the rate of flow of a liquid through a horizontal capillary tube and concluded that the volume V of the liquid flowing per second through a capillary tube varies

1. Directly as the pressure difference p across the two ends of the tube

$$V \propto p$$

2. Directly as the fourth power of radius r of the tube

$$V \propto r^4$$

3. Inversely as the length l of the tube

$$V \propto 1/l$$

4. Inversely as the coefficient of viscosity of the liquid

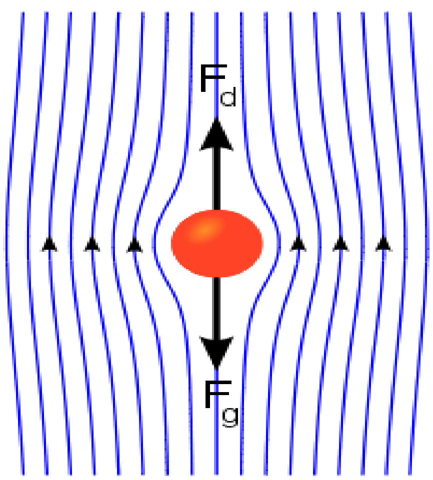
$$V \propto 1/\eta$$

Combining these factors we get

$$V \alpha \frac{pr^4}{\eta l} \quad or \quad V = \frac{\pi}{8}\frac{pr^4}{\eta l}$$

Where π/8 is a constant of proportionality

2.5 STOKE'S LAW



Due to relative motion between different layers of medium, a backward dragging force comes into play which opposes the motion of the body.

Stoke found that the backward dragging force F acting on a small spherical body of radius r, moving through a medium of coefficient of viscosity η, with velocity v is given by

$$F = 6\pi\eta rv$$

This is called Stoke's law of viscosity

2.5.1 Importance of Stoke's law

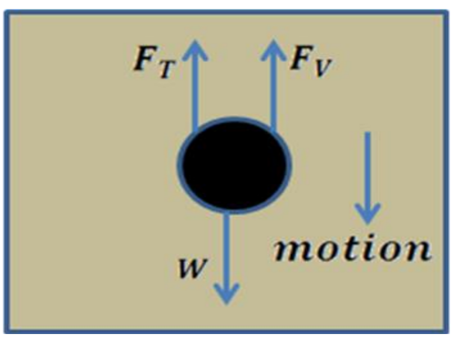
1) It is used in the determination of electron charge with the help of Millikan's experiment.
2) It accounts the formation of clouds.
3) It accounts why the speed of rain drops is less than that of a body falling freely with a constant velocity from the height of clouds.
4) It helps a man coming down with the help of a parachute.

2.5.2 Terminal velocity

Maximum constant velocity acquired by the body while falling freely in a viscous medium is called its terminal velocity.

Forces acting on a spherical body falling freely in a viscous medium are:

1. Weight of the body acting vertically downwards.
2. Upward thrust due to buoyancy equal to weight of liquid displaced.
3. Viscous drag acting in the direction opposite to the motion of body



Let ρ = density of the material of the spherical body

r = radius of spherical body

σ = density of the medium

v = terminal velocity of the body

$$\textit{True weight of the body}, W = \textit{volume} \times \textit{density} \times g$$

$$= \frac{4}{3}\pi r^3 \rho g$$

$$\textit{Upward thrust due to buoyancy}, F_T = \textit{weight of the medium displaced}$$

$$F_T = \textit{volume of the medium displaced} \times \textit{density} \times g$$

$$= \frac{4}{3}\pi r^3 \sigma g$$

$upward\ viscous\ drag, F_V = 6\pi\eta rv$

When body attains terminal velocity, then

$$F_T + F_V = W$$

$$\therefore \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g$$

$$6\pi\eta rv = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$or\ v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

2.5.3 Variation of viscosity

i. ***With increase in temperature***, the viscosity of liquid decreases and the viscosity of all gases increases.

ii. ***With increase in pressure***, the viscosity of liquids increases but the viscosity of water decreases, whereas the viscosity of gases remains unchanged.

2.5.4 Practical uses of the knowledge of viscosity

1) The knowledge of viscosity and its variation with temperature helps us to select a suitable lubricant for a given medicine.
2) The knowledge of viscosity helps us in determining the shape and molecular weight of organic liquids such as proteins, cellulose etc.
3) At railway terminals, the liquid of high viscosity are used as buffers
4) Motion of some instruments is dumped by using the viscosity of air or liquid.
5) The knowledge of viscosity helped Millikan in determining charge on an electron.
6) The phenomenon of viscosity plays an important role in the circulation of blood through arteries and veins of human body.

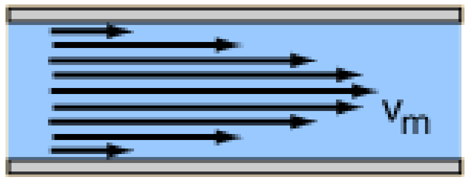
2.6 STREAMLINE FLOW

The flows in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point is called streamline flow.

Stream line: The actual path followed by the procession of particles in a steady flow, which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point is called stream line.

Tube of flow: The bundle of streamlines having the same velocity of the liquid particles over any cross section perpendicular to the direction of flow of liquid is called tube of flow.

2.7 LAMINAR FLOW



If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called laminar flow.

In this flow the velocity of liquid flow is always less than the critical velocity of the liquid. In general, laminar flow is a streamline flow.

2.8 TURBULENT FLOW

Fluid flow in which the fluid undergoes irregular fluctuations or mixing is called turbulent flow. The speed of the fluid at a point is continuously undergoing changes in magnitude and direction, which results in swirling and eddying as the bulk of the fluid moves in a specific direction.

Examples of turbulent flow are:

1. Smoke rising from a cigarette.
2. Flow over a golf ball.
3. The mixing of warm and cold air in the atmosphere by wind, which causes clear-air turbulence experienced during airplane flighting (the blurring of images seen through the atmosphere.)
4. Most of the terrestrial atmospheric circulation
5. The oceanic and atmospheric mixed layers and intense oceanic currents.
6. The external flow over all kind of vehicles such as cars, airplanes, ships and submarines.
7. In windy conditions, trucks that are on the motorway get buffeted by their wake.

2.9 CRITICAL VELOCITY

The critical velocity is that velocity of liquid flow, up to which its flow is streamlined and above which its flow becomes turbulent.

The critical velocity of a liquid flowing through a tube depends upon

1) Coefficient of viscosity of liquid
2) Density of liquid

3) Radius of the tube

2.10 REYNOLD NUMBER

A pure number which determines the nature of flow of liquid through a pipe is called Reynold number.

According to Reynold, the critical velocity of a liquid flowing through a tube of diameter D is given by

$$v_c = \frac{N_R \eta}{\rho D} \quad or \quad N_R = \frac{\rho D v_c}{\eta}$$

Where η = coefficient of viscosity of the liquid,

ρ = density of liquid

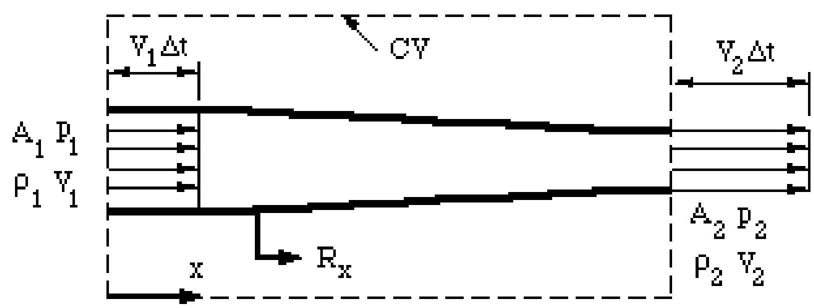
N_R = constant called Reynold number

The Reynolds number is important in analyzing any type of flow when there is substantial velocity gradient (i.e., shear.) It indicates the relative significance of the viscous effect compared to the inertia effect. The Reynolds number is proportional to inertial force divided by viscous force.

If the value of Reynolds number lies between 0 to 2000, the flow of liquid is stream line or laminar. For value of Reynolds number above 3000 the flow of liquid is turbulent and for values of Reynolds number between 2000 to 3000, the flow of liquid is unstable changing from streamline to turbulent flow.

2.11 EQUATION OF CONTINUITY

Consider the steady flow of fluid through a duct. The inflow and outflow are one-dimensional, so that the velocity V and density ρ are constant over the area A



Now we apply the principle of mass conservation.

Volume flow in over $A_1 = A_1 V_1 \Delta t$

Volume flow in over $A_2 = A_2 V_2 \Delta t$

Therefore mass in over $A_1 = \rho A_1 V_1 \Delta t$

Mass in over $A_2 = \rho A_2 V_2 \Delta t$

So $\rho A_1 V_1 = \rho A_2 V_2$

This is a statement of the principle of mass conservation for a steady, one-dimensional flow, with one inlet and one outlet. This equation is called the continuity equation for steady one-dimensional flow

2.12 ENERGIES OF A FLUID

Pressure energy per unit volume = P

Pressure energy per unit mass = P/ρ

Kinetic energy per unit volume = $\frac{1}{2}\rho v^2$

Kinetic energy per unit mass = $\frac{v^2}{2}$

Potential energy per unit volume = ρ gh

Potential energy per unit mass = gh

Pressure head = $\frac{P}{\rho g}$

Velocity head = $\frac{v^2}{2g}$

Gravitational head = h

2.13 BERNOULLI'S THEOREM

According to Bernoulli's theorem for the streamline flow of an ideal liquid, the total energy (the sum of the pressure energy, potential energy and kinetic energy) per unit mass remains constant at every cross-section throughout the flow.

Bernoulli's theorem is an outcome of the principle of conservation of energy applied to a liquid in motion.

Consider a tube of varying cross-section through which an ideal liquid is in streamline flow.

Let P_1, P_2 = pressure applied on the liquid at A and B

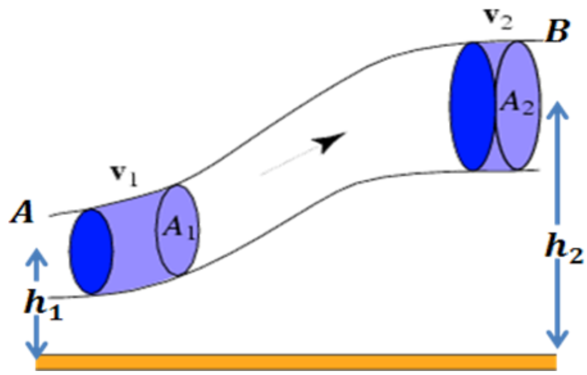
A_1, A_2 = area of cross section of the tube at A and B

h_1, h_2 = mean height of section A and B from the ground

v_1, v_2 = normal velocity of the liquid of flow at A and B

ρ = density of the ideal liquid flowing through the tube

m = mass of the liquid crossing per second through any section of the tube.



According to the equation of continuity

$A_1v_1\rho = A_2v_2\rho = m$ or $A_1v_1 = A_1v_1 = m/\rho = V$

Now force on the liquid at A = P_1A_1

And force on the liquid at section B = P_2A_2

Work done/second on the liquid at A = $P_1A_1 \times v_1 = P_1V$

Work done/second on the liquid at B= $P_2A_2 \times v_2 = P_2V$

Net work done/second on the liquid by the pressure energy = $P_1V - P_2V$

Increase in P.E. per second of the liquid from A to B = $mgh_2 - mgh_1$

Increase in K.E. per second of the liquid from A to B = $(1/2)\ mv_2^2 - (1/2)\ mv_1^2$

Work done/sec by the pressure energy = increase in P.E./sec + increase in K.E./sec

$$P_1V - P_2V = (mgh_2 - mgh_1) + \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)$$

$$or \;\; P_1V + mgh_1 + \frac{1}{2}mv_1^2 = P_2V + mgh_2 + \frac{1}{2}mv_2^2$$

Dividing throughout by m, we get

$$\frac{P_1V}{m} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2V}{m} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2 \qquad \left[Since \;\; \rho = \frac{m}{V}\right]$$

$$Hence \;\; \frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{a constant}$$

Now P/ρ = pressure energy per unit mass

gh = potential energy per unit mass

$(1/2)\ v^2$ = Kinetic energy per unit mass

2.13.1 Limitations of Bernoulli's theorem

1. Bernoulli's equation is based on the assumption that velocity of every particle of liquid across any cross section of tube is uniform. Practically it is not correct. Particles of the innermost layer of liquid have maximum velocity and those on a layer in contact of the tube have least velocity. Therefore, we should take the mean velocity of the liquid.
2. The viscous drag of the liquid which comes into play when the liquid is in motion has not been taken into account.
3. While deriving the Bernoulli's equation it is assumed that there is no loss of energy. Infact some Kinetic Energy is converted into heat and is lost.
4. When a fluid moves in a curved path, the energy corresponding to centrifugal force should also be taken into consideration.

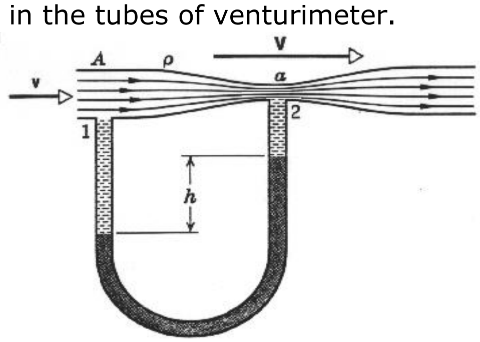
2.13.2 Applications Bernoulli's theorem

1) Atomizer or sprayer
2) Blowing off the roofs during storm
3) Lift on an aeroplane wing
4) Curved motion of spinning ball
5) Collision of two parallel boats moving in same direction.

2.13.3 Venturimeter

Venturimeter is a device for measuring the flow along a pipe.

Construction: It consists of two identical coaxial vide tubes connected by a narrow coaxial tube. A monometer in the form of U tube is attached with one arm at the wider neck and the other arm at the narrow tube middle tube. A low density tube is used in monometer which may not mix with the liquid flowing in the tubes of venturimeter.



Working and theory: Connect this venturimeter horizontally to the pipe through which the liquid is flowing with steady flow. Let the difference of height of liquid column in two arms of U tube is h.

Let ρ = density of liquid flowing through the pipe

ρ_m = density of liquid in U tube

v_1, v_2 = velocity of liquid flow through A and B

P_1, P_2 = Pressure at A and B;

V = volume of liquid flowing per second

According to equation of continuity

$$V = a_1 v_1 = a_2 v_2$$

$$\therefore \frac{a_1}{a_2} = \frac{v_2}{v_1} \text{ and } v_1 = \frac{V}{a_1};\ v_2 = \frac{V}{a_2}$$

$$\text{Using Bernoulli'sequation}, P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$or\ P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right) = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right)$$

$$\text{Pressure difference},\ P_1 - P_2 = h\rho_m g$$

$$\therefore h\rho_m g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right)$$

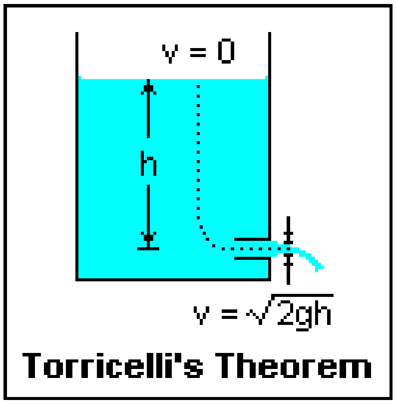
$$or\ v_1 = \sqrt{\frac{2h\rho_m g}{\rho}}\left(\frac{a_1^2}{a_2^2} - 1\right)^{-\frac{1}{2}}$$

$$\text{Volume per sec}, V = a_1 v_1 = a_1 a_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^2 - a_2^2)}}$$

2.13.4 Torricelli's theorem

According to this theorem the velocity with which the liquid flows out of an orifice is equal to that which a freely falling body would acquire in falling through a vertical distance equal to depth of orifice below the free surface of liquid.

The pressure will be the same because they are open to the atmosphere. The fluid velocity at region 2 is much slower than the fluid velocity at region 1. Therefore the fluid velocity at region 2 is negligible. The value of y_1=0. The equation simplifies to



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\frac{1}{2}\rho v_1^2 = \rho g y_2$$

$$v_1 = \sqrt{2gh}$$

2.14 SURFACE TENSION

2.14.1 Adhesive force

It is the force of attraction acting between molecules of two different materials. For example, the force acting between the molecules of water and glass.

2.14.2 Cohesive force

It is the force of attraction acting between molecules of the same material. For example, the force acting between the molecules of water or mercury etc.

1. If cohesive force is larger than the adhesive force, the liquid will not stick to the vessel containing it. It is so in case of mercury and glass.
2. If cohesive force is less than the adhesive force, the liquid will stick to the vessel containing it. It is so in case of water and glass.
3. Cohesive or adhesive force varies inversely as the eighth power of distance between the molecules i.e. [F_c or $F_a \propto 1/r^8$]

2.14.3 Molecular range

It is the maximum distance up to which a molecule can exert some measurable attraction on other molecules. The order of molecular range is 10^{-9}m in solids and liquids.

2.14.4 Sphere of influence

It is an imaginary sphere drawn with a molecule as centre and molecular range as radius. All the molecules in this sphere attract the molecule at the centre and vice-versa.

2.14.5 Surface film

It is the top most layer of liquid at rest with thickness equal to molecular range.

2.14.6 Surface tension

It is the property of the liquid by virtue of which the free surface of the liquid at rest tends to have the minimum surface area and as such it behaves as if covered with a stretched membrane.

Quantitatively, surface tension of a liquid is measured as the force acting per unit length of a line imagined to be drawn tangentially anywhere on the free surface of the liquid at rest. It acts at right angles to this line on both the sides and along the tangent to the liquid surface i.e. $S = F/l$.

Surface tension of a liquid is also defined as the amount of work done in increasing the free surface of liquid at rest by unity at constant temperature

i.e. $S = W / A$.

or $W = S \times A$ = surface tension x area of liquid surface formed.

Surface tension is a molecular phenomenon and it arises due to electromagnetic forces. The explanation of surface tension was first given by Laplace.

S.I. Units of surface tension is Nm^{-1} or Jm^{-2} and c.g.s. unit is dyne cm^{-1} or erg cm^{-2}.

Dimensional formula of surface tension = $[M^1L^0T^{-2}]$

Surface tension is a scalar quantity as it has no specific direction for a given liquid. Surface tension does not depend upon the area of the free surface of liquid at rest.

2.14.7 Surface energy

It is defined as the amount of work done against the force of surface tension in forming the liquid surface of a given area at a constant temperature

i.e. Surface energy = work done = S.T. x surface area of liquid

The S.I. unit of surface energy is joule and cgs unit is erg.

When small drops combine together to form a big drop, the surface area decreases, so surface energy decreases. Hence the energy is released. If this energy is taken by drop, the temperature of drop increases. When a big drop is splitted into number of smaller drops, the surface area of drops increases. Hence surface energy increases. So energy is spent.

2.14.8 Work done in blowing a liquid drop or soap bubble

Work done in forming a liquid drop of radius R, surface tension S is,

$$W = 4\pi R^2S.$$

Work done in forming a soap bubble of radius R, surface tension S is,

$$W = 2 \times 4 \pi R^2S = 8 \pi R^2S$$

Work done in increasing the radius of a liquid drop from r_1 to r_2 is,

$$W = 4 \pi S (r_2^2 - r_1^2)$$

Work done in increasing the radius of a soap bubble from r_1 to r_2 is,

$$W = 8 \pi S (r_2^2 - r_1^2)$$

2.14.9 Formation of a bigger drop by a number of smaller drops

When n numbers of smaller drops of liquid, each of radius r, surface tension S are combined to form a bigger drop of radius R, then

Volume of bigger drop = volume of n smaller drop i.e. $(4/3)\,\pi R^3 = n \times (4/3)\,\pi r^3$ or $R = n^{1/3} r$

The surface area of bigger drop = $4 \pi R^2 = 4 \pi n^{2/3} r^2$

It is less than the area of n smaller drops.

In this process energy is released, given by

$$W = S \times [4 \pi r^2 n - 4 \pi R^2] = 4 \pi S r^2 n^{2/3}(n^{1/3} - 1)$$

$$= 4 \pi S R^2 (n^{1/3} - 1) = 4 \pi S R^3 \left[\frac{1}{r} - \frac{1}{R}\right]$$

a) Excess of pressure inside a liquid drop,

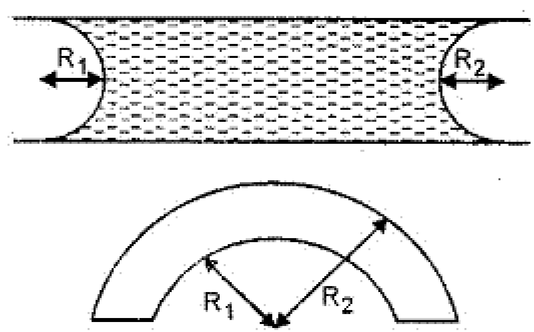
$p = 2\,S/R$

b) Excess of pressure inside a soap bubble,

$p = 4\,S/R$

Where S is a surface tension and R is the radius of the drop or bubble.

2.14.10 Pressure difference (p) across curved surfaces of radii of curvature R_1 and R_2



a) If the curvatures are in mutually opposite direction as shown in Figure

$$p = S\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

b) If the curvatures are in the same direction as shown in Figure

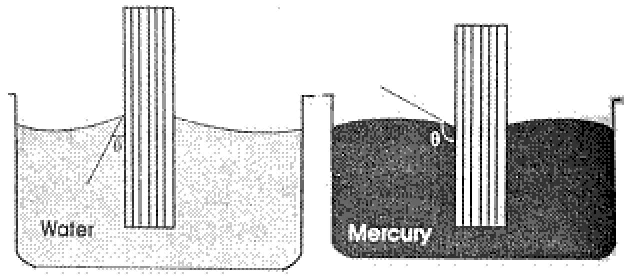
$$p = S\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

c) For a cylindrical surface, $p = S/R$, because $R_1 = R$ and $R_2 = \infty$.

d) For a spherical surface, $p = 2S/R$, because $R_1 = R_2 = R$.

2.14.11 Angle of contact

The angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.



The angle of contact depends upon

a) The nature of solid and the liquid in contact

b) The given pair of the solid and the liquid

c) The impurities

The angle of contact does not depend upon the inclination of the solid in the liquid.

The value of angle of contact (θ) lies between $0^o$and 180^o. For pure water and glass, $\theta = 0^o$. For ordinary water and glass, $\theta = 8^o$. For silver and pure water $\theta = 90^o$. For alcohol and clean glass, $\theta = 0^o$.

The value of the angle of contact is less than 90^o for a liquid which wets the solid surface and is greater than 90^0 if a liquid does not wet the solid surface.

Angle of contact is independent of the angle of inclination of the wall in contact with liquid.

The increase in temperature increases the angle of contact.

The angle of contact increases with the addition of soluble impurities in the liquid.

The addition of detergent in water decreases both the angle of contact as well as surface tension.

The materials used for water proofing increases the angle of contact as well as surface tension.

If a liquid wets the sides of containing vessel, then the value of angle of contact is acute *i.e.* less than 90^o. If a liquid does not wet the sides of containing vessel, then the value of angle of contact is obtuse *i.e.* greater than 90^o.

2.14.12 Capillary action or capillarity

It is the phenomenon of rise or fall of liquid in a capillary tube. The root cause of capillarity is the difference of pressure on the two sides of liquid meniscus in the capillary tube.

The height h through which a liquid will rise in a capillary tube of radius r which wets the sides of the tube will be given by

$$h = \frac{2\,S\cos\theta}{r\rho g} = \frac{2\,S}{R\rho g}$$

Where S is the surface tension of liquid, θ is the angle of contact, ρ is the density of liquid and g is the acceleration due to gravity. R is the radius of curvature of liquid meniscus.

If $\theta < 90^o$, $\cos\theta$ is positive, so h is positive i.e. liquid rises in a capillary tube.

If $\theta > 90^o$, $\cos\theta$ is negative, so h is negative i.e. liquid falls in a capillary tube.

If a capillary tube is of insufficient length as compared to height to which liquid can rise in the capillary tube, then the liquid rises up to the full length of capillary tube but there is no overflowing of the liquid in the form of fountain. It is so because the liquid meniscus adjusts its radius of curvature so that hR = a constant i.e. h R = h' R'.

The height of the liquid column in a capillary tube on the surface of moon is six times than that on the earth.

Rise of liquid in a capillary tube does not violate law of conservation of energy.

2.14.13 Dependence of surface tension

a) On temperature: The surface tension of liquid decreases with rise of temperature. Surface tension of a liquid at its critical temperature is zero. Surface tension of molten cadmium increases with the increase in temperature.

b) On impurities:

1) A highly soluble substance like sodium chloride (common salt) when dissolved in water increases the surface tension of water.

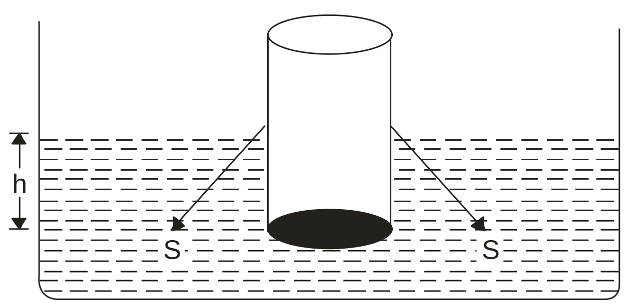
2) When a sparingly soluble substance like phenol, dissolved in water, reduces the surface tension of water.

3) When a detergent or soap is mixed with water, the surface tension of water decreases. Soap or detergent helps in better cleaning of clothes because when it is added to water, reduces the surface tension of water.

c) On electrification: The surface tension of the liquid decreases due to electrification, because a force starts acting outwards normally to the surface of the liquid. It is due to this reason that the soap bubble expands when given positive or negative charge.

d) On contamination: The presence of dust particles or lubricating materials on the liquid surface decreases its surface tension.

Consider a cylindrical glass tube closed at one end. It is made heavy at closed end so that it floats in the vertical position as shown in Figure. If the depth of heavy end is h below liquid surface; then



$$h = \frac{2\pi rS + mg}{\pi r^2 \rho g}$$

2.14.14 Radius of the new bubble formed when two bubbles coalesce

Consider two soap bubbles of radii r_1 and r_2 respectively. If V_1 and V_2 are the volumes of two soap bubbles, then

$$V_1 = \frac{4}{3}\pi r_1^3 \text{ and } V_2 = \frac{4}{3}\pi r_2^3$$

Let S be the surface tension of the soap solution. If P_1 and P_2 are the excess of pressure inside the soap bubbles, then

$$P_1 = 4S/r_1 \text{ and } P_2 = 4S/r_2$$

Let r be the radius of the new soap bubble formed when two soap bubbles coalesce under isothermal conditions. If V and P be the volume and excess of pressure inside this new soap bubble, then

$$V = \frac{4}{3}\pi r^3 \text{ and } P = \frac{4S}{r}$$

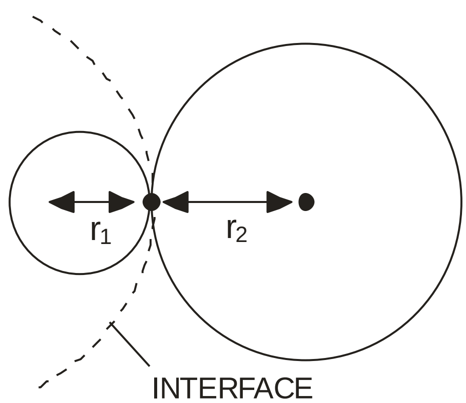
As the new bubble is formed under isothermal conditions, so Boyle's law holds good i.e.

$$P_1V_1 + P_2V_2 = PV$$

$$\frac{4S}{r_1} \times \frac{4}{3}\pi r_1^3 + \frac{4S}{r_2} \times \frac{4}{3}\pi r_2^3 = \frac{4S}{r} \times \frac{4}{3}\pi r^3$$

$$r_1^2 + r_2^2 = r^2 \quad or \quad r = \sqrt{r_1^2 + r_2^2}$$

2.14.15 Radius of interface when two soap bubbles of different radii are in contact



Consider two soap bubbles of radii r_1 and r_2 in contact with each other as shown in Figure. Let r be the radius of the common boundary. If $P_1$and P_2 are the excess pressures on the two sides of the interface then the resultant excess pressure is

$$P = P_1 - P_2$$

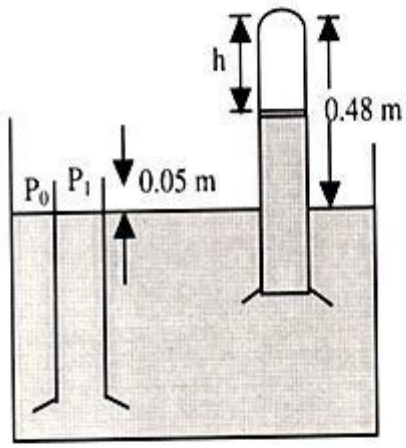
$$\frac{4S}{r} = \frac{4S}{r_1} - \frac{4S}{r_2}$$

or $$\frac{1}{r} = \frac{1}{r} - \frac{1}{r_2}$$

or $$r = \frac{r_1 r_2}{r_2 - r_1}$$

2.15 Solved Examples

Example 1: A wide tube with one end open is immersed vertically in mercury in such a way that a length of 0.05 m extends above the mercury level. The open end of the tube is then closed and the tube is raised further by 0.43m. Calculate the length of the air column above the mercury level in the tube.



Solution: Let the cross-section of the tube be A. When it is open

$P_1 = p_0$ and $V_1 = A \times 0.05$

When tube is closed and 0.43m of it is taken out of mercury, h is the length of the air column above mercury and p_2 the pressure of air inside it,

$p_0 = p_2 + (0.48-h)\rho g$ and $V_2 = Ah$

(as pressure at same level is same)

Applying Boyle's law

$p_1V_1 = p_2V_2$

or, $p_0 \times A \times 0.05 = Ah[p_0-(0.48-h)\rho g]$

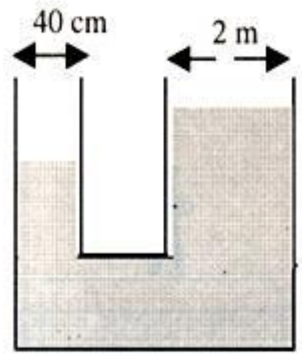
or, $h^2+0.28h-0.038=0$ [taking $p_0=0.76\rho g$](1atm=760mm of Hg)

or, $h = (-0.28 \pm \sqrt{((0.28)^2 + 4 \times 0.038)})/2$

$h \approx 0.1m$

Example 2: A liquid is filled inside a vessel as shown. Radii of the left and right column of the vessel are 20cm and 1m respectively. On the left side if man of 60 kg stands on the surface of liquid, what mass can be lifted on the other side.

Solution: Area of cross section of the left column = $\pi\ (20\times10^{-2})^2 m^2 = a$.
Area of cross section of the right column = $\pi\ (1)^2 m^2 = A$
If man of mass 60 kg stands on the left column,



Pressure on left column = $(60\times(10))/a\ N/m^2$

$$= 600/(\pi\times4\times10^{-2})\ N/m^2$$

[Considering $g = 10\ m/s^2$]

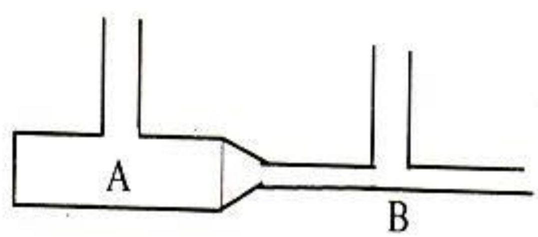
According to Pascal's law the same pressure reaches right column.

Force on the surface of right column= Pressure x Area

$= [600/(\pi\times4\times10^{-2})] \times \pi \times (1)^2 = 150\times10^2\ N$

Mass which can be lifted= $(150\times10^2)/g = 15\times10^2$ kg = 1500 kg.

Example 3: Let a liquid enter a tube of radius 2 cm with velocity 10 cm/s. Subsequently it enters another connected tube of radius 1cm. Two manometer tubes are attached to these connected tubes of different cross section (as shown in the figure). Find the difference in heights of the levels of water in manometer tubes.



Solution: Applying equation of continuity for the two cross-sections we get $A_AV_A=A_BV_B$ (if density of the liquid is constant)
Now, $A_A = \pi r_A^2$; $A_B=\pi r_B^2$; V_A = 10 cm/s.
V_B = velocity of liquid in cross section of radius 1cm.

$= (\pi r_A^2)/(\pi r_B^2).V_A = (4/1)10 = 40$ cm/s.

Applying Bernoulli's theorem to points A and B which are at same horizontal level.

$$P_A + 1/2\,\rho V_A^2 = P_B + 1/2\,\rho V_B^2$$

or, $P_A - P_B = 1/2\rho(V_B^2 - V_A^2)$

But $P_A - P_B = \rho g h_A - \rho g h_B = \rho g(h_A - h_B)$

$\rho g(h_A - h_B) = 1/2\rho(V_B^2 - V_A^2)$

Or, $(h_A - h_B) = (V_B^2 - V_A^2)/2g$ = difference in levels

$= (40^2 - 10^2)/2\times980 = (1600-100)/1960 \approx 0.76$cm.

Example 4: If the terminal speed of a sphere of gold (density = 19.5 kgm^{-3}). Find the terminal speed of a sphere of silver (density = 10.5kgm^{-3}) of the same size in the same liquid.

Solution: $V_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

Where
ρ = density of substance of body
σ = density of liquid
From given data

$$\frac{V_{T(Ag)}}{V_{t\,(Gold)}} = \frac{\rho_{Ag} - \sigma_1}{\rho_{Gold} - \sigma_1}$$

$\Rightarrow$ $V_T(Ag) = \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2$

$= \frac{9}{18} \times 0.2 = 0.1\ ms^{-1}$

Example 5: A cylinder of height 20m is completely filled with water. What is the velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom?

Solution: $p_0 + \rho g h = p_0 + \frac{\rho v^2}{2}$

Or $v = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 20}$

= 20 ms^{-1}

Example 6: A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to

(a) $L/\sqrt{2\pi}$ (b) $2\pi L$

(c) L (d) $L/2\pi$

Solution: Volume of water flowing out per second from both the holes are equal.

$\therefore$ $a_1 v_1 = a_2 v_2$

or $(L^2)\sqrt{2h(y)} = \pi R^2 \sqrt{2g(4y)}$

or $R = \frac{L}{\sqrt{2\pi}}$