COMPOUND INTEREST

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3.1.2 Compound Interest

3.1.2.1 Basic Definition and Concepts

Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say yearly or half-yearly or quarterly to settle the previous account.
In such cases, the amount after first unit time becomes the principal for the second unit. The amount after second unit becomes the principal for the third unit and so on.
After a specified period, the difference between the original principal and the final amount is called **Compound Interest (C.I)**

$$C.I = \text{Principal } [(1 + \frac{Rate}{100})^{\text{Time}} - 1]$$

3.1.2.2 Compounding For Different Time Periods and Rates

Let principal = Rs. P, Time = T yrs and Rate = R% per annum

- When interest is compounded annually, the

  $$\text{Amount} = P\,[1+\frac{R}{100}]^{T}$$

Example: Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.

Solution: Amount= Rs. $[7500 \times \left(1 + \frac{4}{100}\right)^2]$ = $(7500\times \frac{26}{25}\times\frac{26}{25})$ = Rs. 8112.

$\therefore$ C.I. = Rs. (8112 - 7500) = Rs. 612.

- When interest is compounded half-yearly, then the

  $$\text{Amount} = P\left[1+\frac{\frac{R}{2}}{100}\right]^{2T}$$

Example: Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly.

Solution: Principal = Rs. 10,000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

$\therefore$ Amount = Rs $[10000 \times (1+\frac{2}{100})^4]$ = Rs. $(10000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50})$
= Rs. 10824.32.

$\therefore$ C.I. = Rs. (10824.32 - 10000) = Rs. 824.32.

- When interest is compounded quarterly ,then the

  $$\text{Amount } P\left[1+\frac{\frac{R}{4}}{100}\right]^{4T}$$

- When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years , then the

  $$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{{}^2/_5\,R}{100}\right) \quad \text{(approximately)}$$

Example: Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

Solution: Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$ years.

Amount = Rs. $\left[8000 \times \left(1+\frac{15}{100}\right)^2 \times \left(1+\frac{^1/_3 \times 15}{100}\right)\right]$

= Rs. $(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20})$ = Rs. 11109.

C.I. = Rs. (11109 - 8000) = Rs. 3109.

- If Principal (P) = Re 1, then the C.I for some frequently used interest rates and time periods is summarized below:

Time	1 Year	2 Years	3 Years
R	$\left(1+\frac{R}{100}\right)$	$\left(1+\frac{R}{100}\right)^2$	$\left(1+\frac{R}{100}\right)^3$
10	$\frac{11}{10}$	$\frac{121}{100}$	$\frac{1331}{1000}$
5	$\frac{21}{20}$	$\frac{441}{400}$	$\frac{9261}{8000}$
4	$\frac{26}{25}$	$\frac{676}{625}$	$\frac{17576}{15625}$

3.1.2.3 Some Specific Cases

- When rate of interest is R_1%, R_2% and R_3% for 1st year, 2nd year and 3rd year respectively,
  Then Amount = P $[1+\frac{R_1}{100}] \times [1+\frac{R_2}{100}] \times [1+\frac{R_3}{100}]$
- Present worth of Rs. x due T years hence is given by :
  Present Worth = $\frac{x}{\left(1+\frac{R}{100}\right)^T}$.
- If a sum becomes x times in y years at CI then it will be $(x)^n$ times in ny years.

Example: A sum of money at compound interest amounts to thrice itself in there years. In how many years will it be 9 times itself?

Solution: If a sum becomes 3 times in 3 years it will be $(3)^2$ times in $2 \times 3 = 6$ years.

- If a certain sum becomes 'n' times in 'T' years, the rate of compound interest R is equal to $100[(n)^{1/T} - 1]$

Example: At what rate per cent compound interest does a sum of money become nine-fold in 2 years?

Solution: $R = 100[(9)^{1/2} - 1] = 100(3-1) = 200\%$

Example: At what rate percentage (compound interest) will a sum of money become eight times in three years?

Solution: Rate % = $[(8)^{1/T} - 1] \times 100$

$= [(8)^{1/3} - 1] \times 100 = (2-1) \times 100 = 100\%$

- If a sum 'P' becomes 'Q' in t_1 years at compound rate of interest, then after t_2 years the sum becomes Rs. $\frac{(Q)^{T_2/T_1}}{(P)^{T_2/T_1 - 1}}$

Example: Rs 4800 becomes Rs 6000 in 4 years at a certain rate of compound interest. What will be the sum after 12 years?

Solution: Amount = $\frac{(6000)^{12/4}}{(4800)^{12/4-1}} = \frac{(6000)^3}{(4800)^2}$ = Rs. 9375

- If the rate of Interest is R_1% for the first year, R_2% for the second year …. and R_T% for the T_{th} year, the compound interest on Rs x for T years is given by:

  $$x\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right).....\left(1+\frac{R_T}{100}\right) - x$$

Example: Find the compound interest on Rs 10000 for 3 years if the rate of interest is 4% for the first year, 5% for the second year and 6% for the third year.

Solution: Compound interest = $10000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{6}{100}\right)$ - 10000

$= 10000\left(\frac{26}{25}\right)\left(\frac{21}{20}\right)\left(\frac{53}{50}\right)$ - 10000

= 11,575.20 – 10,000 = Rs 1,575.2

- If the compound rate of interest for the first T_1 years is R_1%, for the next T_2 years is R_2%, for the next T_3 years is R_3%, ….. and the last T_n years is R_n%, then compound interest on Rs x for $(T_1+T_2+T_3+\ldots\ldots T_n)$years is

  $$\left[x\left(1+\frac{R_1}{100}\right)^{T_1}\left(1+\frac{R_2}{100}\right)^{T_2} \ldots\ldots\left(1+\frac{R_n}{100}\right)^{T_n}\right] - x$$

- If a man borrows Rs. P at R% compound interest and pays back Rs. A at the end of each year, then at end of the nth year he should pay Rs.

  $$P\left[1+\frac{R}{100}\right]^n - A\left[\left(1+\frac{R}{100}\right)^{n-1} + \left(1+\frac{R}{100}\right)^{n-2} + \ldots\ldots + \left(1+\frac{R}{100}\right)^1\right]$$

Example: A man borrows Rs 3000 at 10% compound rate of interest. At the end of each year he pays beck Rs 1000. How much amount should he pay at the end of the third year to clear all his dues?

Solution: Amount = $3000\left[1+\frac{10}{100}\right]^3 - 1000\left[\left(1+\frac{10}{100}\right)^2 + \left(1+\frac{10}{100}\right)^1\right]$

$= 3000\left(\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\right) - 1000\left[\left(\frac{11}{10}\right)^2+\frac{11}{10}\right]$

$= 3993 - \left\{1000\times\frac{121}{100}+1000\times\frac{11}{10}\right\}$

= 3993 – 1210 – 1100 = Rs. 1683

3.1.2.4 Cases Relating Simple and Compound Interest

- Simple Interest = $\frac{RT}{100\left[\left(1+\frac{R}{100}\right)^T-1\right]} \times \text{Compound Interest}$

Example: If the compound interest on a certain sum for 2 yrs at 3% be Rs.101.50, what would be the simple interest?

Solution: Simple Interest = $\frac{3\times2}{100\left[\left(1+\frac{3}{100}\right)^2-1\right]} \times 101.50$ = Rs. 100

Example: The compound interest on a certain sum of money for 2 years at 10% per annum is Rs 420. Find the simple interest at the same rate and for the same time.

Solution: We have S.I = $\frac{RT}{100\left[\left(1+\frac{R}{100}\right)^{T}-1\right]} \times C.I$

When T = 2,

S.I = $\frac{2R}{100[1+\frac{R^2}{100^2}+\frac{2R}{100}-1]} \times$C.I = $\frac{2R\times CI\times 100}{R^2+200R}$

S.I = $\frac{200R}{R(R+200)} \times$C.I

Therefore,

S.I = $\frac{200\times 10\times 420}{10\times 210}$ = Rs 400

- If on a certain sum of money, the S.I for 2 years at the rate R% per annum is Rs X, then the difference in the compound interest and simple interest is given by Rs. $\frac{XR}{200}$. This formula is applicable only for 2 years.

Example: On a certain sum of money, the simple interest for 2 years is Rs. 50 at the rate of 5% per annum. Find the difference in CI and SI

Solution: Difference in CI and SI = $\frac{50\times 5}{200} = Rs.1.25$

- When difference between the CI and SI on a certain sum of money for 2 years at R% rate is x, then the sum is given by

Sum = $\frac{Difference\times 100\times 100}{Rate\times Rate}$ =Difference $\times \left(\frac{100}{R}\right)^2$

Example: The difference between the compound interest and the simple interest on a certain sum of money at 5% per annum for 2 year is Rs. 1.50. Find the sum.

Solution: $\text{Sum} = 1.5(\frac{100}{5})^2 = 1.5\times 400 = Rs.600$

- On a certain sum of money, the difference between compound interest and simple interest for 2 years at R% rate is given by Sum $\times \left(\frac{R}{100}\right)^2$.

Example: Find the difference between the compound interest and simple interest for Rs. 2500 at 10% per annum for 2 years.

Solution: $\text{Difference=Sum}(\frac{R}{100})^2 = 2500\times(\frac{10}{100})^2 = Rs.25$

- If the difference between CI and SI on a certain sum for 3 years at R% is Rs. x, the sum is given by :

$$\text{Sum} = \frac{Difference\times(100)^3}{R^2(300+R)}$$

Example: If the difference between CI and SI on a certain sum of money for 3 years at 5% per annum is Rs. 122, find the sum.

Solution: $\text{Sum} = \frac{122\times 100\times 100\times 100}{5^2(300+5)} = Rs.16{,}000$

- On a certain sum of money, the difference between compound interest and simple interest for 3 years at R% per annum is given by:

$$\text{Difference} = \frac{Sum\times R^2(300+R)}{(100)^3}$$

Example: Find the difference between CI and SI on Rs. 8000 for 3 years at 2.5% per annum.

Solution: $\text{Difference} = \frac{8000\times(2.5)^2\times(300+2.5)}{100\times100\times100}$

$= \frac{8\times25\times25\times3025}{100\times100\times100} = \frac{121}{8} = Rs.15.125$

Some More Examples:

Example: What sum of money at compound interest will amount to Rs 2249.52 in 3 years, if the rate of interest is 3% for the first year, 4% for the second year, and 5% for the third year?

Solution: By the rule of fraction:

Principal = 2249.52 $(\frac{100}{103})$ $(\frac{100}{104})$ $(\frac{100}{105})$ = Rs 2000.

Example: The compound interest on a certain sum for 2 yrs is Rs 40.80 and simple interest is Rs 40.00. Find the rate of interest per annum and the sum.

Solution: [Valid for 2 yrs only]: Rate = $\frac{2\times Difference\ in\ C.I\ and\ S.I}{S.I}\times100$ **(Tip)**

Rate= $\frac{2\times0.8}{40}\times100 = 4\%$

and Sum = $\frac{40\times100}{4\times2}$= Rs 500

Example: If the compound interest on a certain sum for 2 years at 4% p.a. is Rs. 102, what would be the simple interest at the same rate for two years?

Solution: S.I. $= \frac{2\times C.I}{2+\frac{r}{100}}$ **(Tip)**

$= \frac{2\times102}{2+\frac{4}{100}} = Rs.100.$

Example: A sum of money becomes double in 3 years at compound interest compounded annually. In how many years will it become 4 times of itself?

Solution: Time = $\frac{T\log b}{\log a}$, **(Tip)**

Here T = 3, a = 2 and b = 4

$= \frac{3\log4}{\log2}$

$= \frac{3\times2\log2}{\log2}$ = 6 years.

Example: What rate of C.I. for a sum of Rs. 8000 will amount to Rs. 8820 in 2 years if the interest is calculated every year?

Solution: Rate $= \left[(\frac{8820}{8000})^{\frac{1}{2}} - 1\right]\times100\%$ **(Tip)**

Here A = Rs. 8000, B = Rs. 8820 and n = 2

$= \left[(\frac{B}{A})^{\frac{1}{n}} - 1\right]\times100\%$ $= \left[(\frac{441}{400})^{\frac{1}{2}} - 1\right]\times100\%$

$= (\frac{21}{20} - 1)\times100$ = 5%