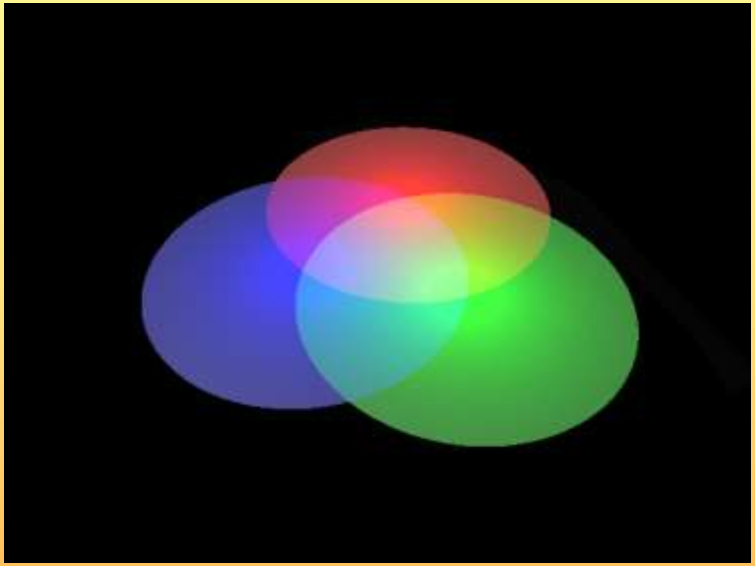


SET THEORY

Subsets

Topic overview - Set Theory

Basics of Sets
- Definition and concept
- Representation of sets
- Types of sets

Subsets
- Subsets
- Subsets of Real numbers
- Power set and Universal set

Operations on sets
- Venn Diagrams
- Union and Intersection of sets
- Difference and Symmetric difference
- Complement of a set

Practical Applications of sets
- Important results on Cardinal numbers
- Practical problems

Subsets

- A set X is said to be a subset of a set Y, if every element of X is also an element of Y. In notation form: $X \subseteq Y$, if $a \in X \Rightarrow a \in Y$.
- If there is at least one element in Y which does not belong to set X, then X is a proper subset of Y and is denoted as $X \subset Y$. Alternately, we can also say that Y is a superset of X.

Note:

1. Every set is a subset of itself, i.e. $A \subseteq A$
2. Empty set ϕ is a subset of every set.

Example: All subsets of the set {1, 2} are : {1}, {2}, {1, 2}, ϕ.

Example: $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}$ but $\mathbf{Z} \not\subset \mathbf{N}$

Example: If A = {2, 4, 6} & B = {x : x is an even Natural number less than 7}
Then, $A \subseteq B$ and also $B \subseteq A$. $\therefore$ A = B.

B.

Subsets

Example: State whether the following statements are true or false.

Solution: $\{x : x \text{ plays a sport}\} \subset \{x : x \text{ plays cricket}\}$ False

$(s : s \text{ is a square}\} \subset \{s : s \text{ is a rectangle}\}$ True

$3 \subset \{1, 2, 3\}$ False. $\{3\} \subset \{1,2,3\}$

$\{x : x+3 = 3\} = \phi$ False. $\{0\} \neq \phi$

Example: If $A = \{u, v, \{w, x\}, y\}$, state whether the following statements are true or false.

Solution: $\{w, x\} \subset A$ False

$\{\{w, x\}\} \subset A$ True

$\{u, v, w\} \subset A$ False

$\{\phi\} \subset A$ False

B.

Subsets

Example: Let A, B, C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$? Show by example.

Solution: Let A = {1}, B = {{1}, 2} and C = {{1}, 2, 3}

Here $A \in B$ and $B \subset C$

But $A \not\subset C$, since $1 \in A$ but $1 \notin C$.

Hence $A \not\subset C$.

Note: An element of a set can not be a subset of the set.

Example: If $A \not\subset B$ and $B \not\subset C$, is it true that $A \not\subset C$? Show by example.

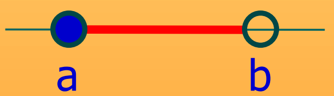
Solution: Let A = {1,2}, B = {2,3} and C = {1, 2, 4}

Here $A \not\subset B$ and $B \not\subset C$ but $A \subset C$

Hence the statement is False.

Intervals as subsets of Real Numbers

Intervals of Real numbers are subsets of **R.** There is a specific notation for representing different type of intervals and show them on a number line.

Set	Interval type	On Number line
$(a, b) = \{x : a < x < b\}$	Open	
$[a, b] = \{x : a \leq x \leq b\}$	Closed	
$(a, b] = \{x : a < x \leq b\}$	Semi-open Or Semi-closed	
$[a, b) = \{x : a \leq x < b\}$		

Thus: Set **R** is represented as $(-\infty, \infty)$

Set of Non-negative Real numbers is represented as $[0, \infty)$

Set $\{x : x \in \mathbf{R}, -3 < x \leq 7\}$ is represented as $(-3, 7]$

Note: ∞ and $-\infty$ are not finite numbers, hence always represented as open ended.

Power Set

- ❖ The set of all possible subsets of a given set is called its Power set. Hence, the elements of the Power set are sets themselves. Power set of a given set A is denoted as P(A). Symbolically, $P(A) = \{S \mid S \subseteq A\}$

Example: If $A = \{1, 2, 3\}$, then its all possible subsets are : ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 2, 3\}$.
Hence $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, A\}$

- ❖ The cardinal number of the power set of a set containing **n** elements = $\mathbf{2^n}$.

Example: If $A = \{1, 2, 3\}$, then $n(A) = 3$. $\therefore n[P(A)] = 2^3 = 8$

Power Set

Example: Show that $n\{P[P(P(\phi))]\} = 4$.

Solution: $P(\phi) = \{\phi\}$

$P[P(\phi)] = \{\phi,\{\phi\}\}$

$P[P(P(\phi))] = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$ ⇒ $n\{P[P(P(\phi))]\} = 4$

Example: Let A = {1,2,3,4}, B = {1,2,3} and C = {2,4}. Find all sets X which satisfy the conditions: $X \subset B$ and $X \subset C$.

Solution: $X \subset B$ and $X \subset C$ ⇒ $X \in P(B)$ and $X \in P(C)$

$P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$P(C) = \{\phi, \{2\}, \{4\}, \{2,4\}\}$

$\therefore X = \phi, \{2\}$

Universal Set

A set with the collection of all possible elements, which can serve as a 'master set' in a given context, is called a Universal set. It is denoted as **U**.

Example: If A = {a, b, c}, B = {b, d, e ,f} and C = {a, c, e, g} then U = {a, b, c, d, e, f, g} can be taken the Universal set.

Example: For **N** (set of Natural numbers), **Z**, **Q, R** or **C,** any of these can serve as Universal set.

Thanks…