# VECTOR ALGEBRA

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- **Directed Line Segment:** Any given portion of a given straight line where for the two points are distinguished as initial and terminal points is called a *directed line segment*. The directed line segment with initial points A and terminal point B is denoted by the symbol $\overrightarrow{AB}$.

- **Length, Support and sense of Directed Line Segment**


*Length* : The length of $\overrightarrow{AB}$ is the distance between A and B, and is denoted by the symbol $|\overrightarrow{AB}|$.

*Support* : The line of unlimited length of which a directed line segment is a part, is called its line of support or simply the support.

*Sense*: The sense of $\overrightarrow{AB}$ is from A to B and that of $\overrightarrow{BA}$ from B to A, so that the sense of a directed line segment is from its initial point to the terminal point.

- **Vector:** A directed line segment is called a vector.

- **Scalar:** Any entity which has only magnitude but no direction is called a scalar.

- **Equality of two Vectors:** Two vectors are said to be equal if they have.
  - the same length;
  - the same or parallel supports; and
  - the same sense.

- **Co-initial Vectors:** Vectors with the same initial point are called co-initial vectors.
- **Zero Vector:** Vector whose initial and terminal points are coincident is called the zero vector.
- **Like Vectors:** Vectors which have the same sense of directions are called 'like vectors'.
- **Unit Vector:** Vector whose length is one unit is called 'unit vector'.
- **Algebra of Vectors:** There are four operations in vectors.

- **Multiplication of Vectors by Scalars:** Let $\vec{a}$ be any given vector and m be any given scalar. Then the symbol $m\vec{a}$ is a vector such that

  $|m\vec{a}| = |m|\,|\vec{a}|$, . ie.,

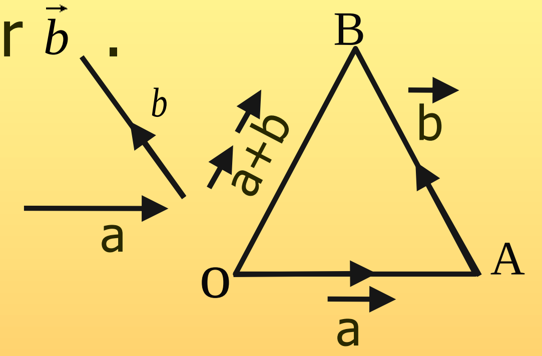
- ✓ the length of $m\vec{a}$ is |m| times the length of $|\vec{a}|$.
- ✓ Support of $m\vec{a}$ is same or parallel to that of $\vec{a}$ .
- ✓ Direction of $m\vec{a}$ is same or opposite to that of $\vec{a}$ according as m is positive or negative.

**Properties** :

- ✓ $(mn)\vec{a} = m(n\vec{a})$
- ✓ $0\vec{a} = \vec{0}$ , so that the product of vector $\vec{a}$ by the zero scalar is the zero vector.

- **Addition of Vectors:** Let $\vec{a}$, $\vec{b}$ be two given vectors. Take a point O. Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{AB} = \vec{b},$ so that the terminal point of the vector $\vec{a}$ is the initial point of the vector $\vec{b}$ .

  Then the vector $\overrightarrow{OB}$ is said to be the sum of the vectors $\vec{a}$ and $\vec{b}$ and we write $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \vec{a} + \vec{b}.$




  In fact, sum of two vectors is independent of the choice of the point O.

- **Negative of a Vector:** The vector which has the same magnitude and same support as that of $\vec{a}$ but the opposite sense, is called the negative of $\vec{a}$ and is denoted by $-\vec{a}.$

- **Linear Combination:** A vector, $\vec{r}$, is said to be linear combination of vectors, $\vec{a}, \vec{b}, \vec{c}$, etc., if there exist scalars x, y, z, etc., such that

$$\vec{r} = x\,\vec{a} + y\,\vec{b} + z\vec{c} + \ldots..$$

- **Collinear Vectors:** Two vectors having the same or parallel supports are called collinear vectors.

  **Note:** If any two vectors are collinear, then each is a scalar multiple of the other.

- **Coplanar Vectors:** Two vectors having the same or parallel supports are called collinear vectors.

  **Result:** If $\vec{a}$ , $\vec{b}$ be two given non-collinear victors, then every vector $\vec{r}$ coplanar with $\vec{a}$ and $\vec{b}$ , can be represented as a linear combination x $\vec{a}$ + y$\vec{b}$, x, y being some scalars. Also this representation is unique.

- **Position Vectors:** The position vector of any point-P with respect to the origin O is the vector $\overrightarrow{OP}$

**Note :** $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \textit{position vector of } B \text{ - } \textit{position vector of } A.$

- **Scalar or Dot product:** The scalar or dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined to be the scalar a b cos θ, where a and b are the moduli of $\vec{a}$ and $\vec{b}$ and θ is the angle between the vectors $\vec{a}$ and $\vec{b}$. It is denoted by $\vec{a}\,.\vec{b}$.

✓ For mutually perpendicular unit vectors, $\hat{i}$ , $\hat{j}$ and $\hat{k}$, we have $\hat{i}.\ \hat{i} = \hat{j}.\ \hat{j} = \hat{k}.\ \hat{k} = 1$, $\hat{i}.\ \hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$.

✓ The scalar product is commutative, i.e., $\vec{a}.\vec{b} = \vec{b}.\vec{a}.$

- **Sign of Scalar product:** If $\vec{a}$ and $\vec{b}$ be two non-zero vectors, then the scalar product $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$ is positive, negative or zero according as the angle between the vectors is acute, obtuse or right.

- **Length of a vector as a scalar product:** If $\vec{a}$ be any vector, then the length $|\vec{a}|$ is the non-negative square root $\sqrt{\vec{a}.\vec{a}}$ of the scalar product $\vec{a}.\vec{a}$ .

  **Note:** $\vec{a}.\vec{a}$ is usually denoted by $\overrightarrow{a^2}$ or by $a^2$.

- **Projection of a vector on a line:** Projection of a vector $\vec{a}$ on a given line is $|\vec{a}| \cos \theta$, where $\theta$ is the angle between the vector $\vec{a}$ and the given line.

- **Angle between two vectors in terms of scalar products:** If θ be the angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ , then

$$\theta = \cos^{-1}\left[\frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}\right] = \cos^{-1}\left[\frac{\vec{a}\,.\,\vec{b}}{\left(+\sqrt{\vec{a}.\vec{a}}\right)\left(+\sqrt{\vec{b}.\vec{b}}\right)}\right]$$

**Properties:**

- If the vectors $\vec{a}$ , $\vec{b}$ are perpendicular, then θ = 90°.
  ∴ $\vec{a} \cdot \vec{b}$ = 0
- $(m\vec{a}.\, n\vec{b}) = mn(\vec{a}.\, \vec{b})$, where $\vec{a}$, $\vec{b}$ are any vectors and m, n any scalars.
- Distributivity : $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ for all vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

- **Right-handed and left-handed vector triads :** Vector $\vec{a}, \vec{b}, \vec{c}$ is said to be right-handed or left-handed triad according as the right handed screw is translated along $\vec{c}$ or opposite to $\vec{c}$, when it is rotated from $\vec{a}$ towards $\vec{b}$, through an angle less than $180°$

- **Vector product or Cross product :** The vector product, denoted by $\vec{a} \times \vec{b}$, of two vectors $\vec{a}, \vec{b}$ taken in this order, is the vector $\vec{c}$, where

➢ $|\vec{c}| = |\vec{a}||\vec{b}| \sin\theta.$

$\theta$ is the angle between the vectors $\vec{a}$ and $\vec{b}$ and $0 \le \theta \le 180°$.

- the support of $\vec{c}$ is perpendicular to supports of $\vec{a}$ and $\vec{b}$,
- the sense of $\vec{c}$ is such that the triad $\vec{a}, \vec{b}, \vec{c}$ forms a right-handed system.

**Note:** The vector product $\vec{a} \times \vec{b}$ is perpendicular to each of the vectors $\vec{a}$, and $\vec{b}$.

- **An important relation:** $\left(\vec{a} \times \vec{b}\right)^2 = \overrightarrow{a^2}\overrightarrow{b^2} - \left(\vec{a} \cdot \vec{b}\right)^2$

**Properties :**

- The vector product of two parallel vectors is the zero vector. Since $\theta = 0$ or $180°$ $\therefore$ $\sin\theta = 0$

- The vector multiplication is not *commutative*. In fact,

$$\vec{a}\times\vec{b} = -\left(\vec{b}\times\vec{a}\right)$$

- **Geometrical interpretation of vector product:** Geometrically, magnitude of the vector product $\vec{a}\times\vec{b}$ , i.e., ab sin $\theta$ is the area of the parallelogram constructed on $\vec{a}$ and $\vec{b}$ as adjacent sides.

- **Scalar triple product :** Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be any three vectors. The scalar product $\vec{a}$ and $\left(\vec{b}\times\vec{c}\right)$, *ie.*, $\vec{a}.\left(\vec{b}\times\vec{c}\right)$ is called the scalar triple product of $\vec{a}$, $\vec{b}$ and $\vec{c}$ in this order and is denoted by $\left[\vec{a}, \vec{b}, \vec{c}\right]$ or $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

**Observations :**

- If $\vec{a}, \vec{b}$ and $\vec{c}$ be any three vectors, then $\left[\vec{a}, \vec{b}, \vec{c}\right] = \left[\vec{b}, \vec{c}, \vec{a}\right] = \left[\vec{c}, \vec{a}, \vec{b}\right]$
- $\left[\vec{a}, \vec{b}, \vec{c}\right] = -\left[\vec{a}, \vec{c}, \vec{b}\right].$
- $\left[\vec{a}, \vec{a}, \vec{b}\right] = 0$

- **Vector triple product :** For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$,

  $$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a}.\vec{c}\right) - \left(\vec{a}.\vec{b}\right)\vec{c}, \ \vec{a} \times \left(\vec{b} \times \vec{c}\right) \neq \left(\vec{a} \times \vec{b}\right) \times \vec{c}$$

- **Vector of a tetrahedron:** The volume of a tetrahedron ABCD is

  $$\frac{1}{6}\left|\overrightarrow{AB} \times \overrightarrow{AC}.\overrightarrow{AD}\right|.$$

Thank You…