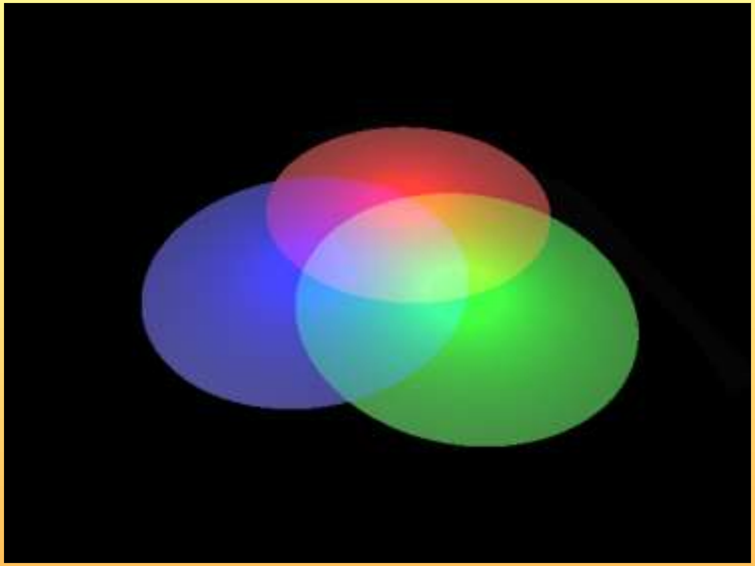


SET THEORY

Basics of sets

Topic overview - Set Theory

Basics of Sets
- Definition and concept
- Representation of sets
- Types of sets

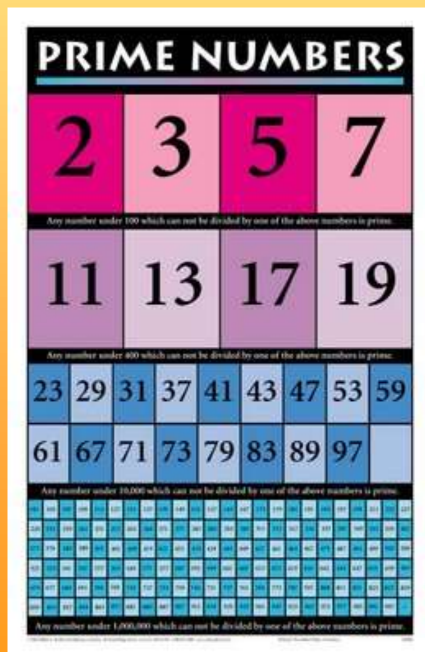
Subsets
- Subsets
- Subsets of Real numbers
- Power set and Universal set

Operations on sets
- Venn Diagrams
- Union and Intersection of sets
- Difference and Symmetric difference
- Complement of a set

Practical Applications of sets
- Important results on Cardinal numbers
- Practical problems

Introduction to sets

In every day life, we come across collection of objects of a particular type such as, members of a football team, a crockery set, a pack of playing cards etc.



So too in Mathematics, we come across collections of objects, such as Odd numbers, Natural numbers, Prime numbers, Polygons etc.

Introduction to sets

- The use of set language allows us to state Mathematical concepts and results in a generalized way with wide applicability.

  For example, rather than stating:
  1+2 = 2+1, 2+3 = 3+2, 3+4 = 4+3, 4+5 = 5+4, …….
  It is much more sensible to state: Commutative property is valid for the set of Natural numbers.

- It is customary nowadays to state all Mathematical concepts and results in the set theoretic language. Sets are especially indispensable in the study of Geometry, Sequences, Probability, Matrices etc.

- Sets are used to define relations and functions, which are a key analytical tool in study of all branches of Science.

Definition of Set

- **A set is a <u>well-defined</u> collection of objects or elements**.

Example: **"**Prime numbers less than 10 (viz. 2, 3, 5, 7)" is a set.

Example: **"**States of India" is a set.

Example: **"**Vowels in English alphabet (viz. a, e, i, o, u)" is a set.

However, "Intelligent boys in a class" is *not a set,* since the term "intelligent" is vague and not well defined.

<u>Note:</u>

- Each element in a set is unique.
- Sets are usually denoted by capital letters A, B, C, X, Y, Z etc.
- Elements of a set are usually denoted by small letters a, b, c, x, y, z etc.
- Greek letter $\in$ (epsilon) denotes an element "belongs to" a set. $\notin$ denotes "does not belong to".

Commonly used Sets in Mathematics

N **:** Set of all Natural numbers.

Z **:** Set of all Integers.

$\mathbf{Z^+}$ **:** Set of all positive Integers.

Q **:** Set of all Rational numbers.

$\mathbf{Q^+}$ **:** Set of all positive Rational numbers.

T **:** Set of all Irrational numbers.

R **:** Set of all Real numbers.

$\mathbf{R^+}$ **:** Set of all positive Real numbers.

C **:** Set of all Complex numbers.

Representation of Sets

➢ Roster or Tabular form

In this form, a set is described by listing elements within braces {} and separated by commas.

Example: Set of positive integer factors of 24 is {1, 2, 3, 4, 6, 8, 12, 24}

Example: Set of alphabets in word ABACUS is {A, B, C, U, S}

Note:
1. Order of listing elements does not matter.
2. Each element is written only once even if it occurs more than once.

➢ Set-Builder form

In this form, a set is defined by specifying the unique, common characteristic of all the elements of the set.

Example: P = {p | p is a positive integer factor of 24}
A = {a : a is an alphabet in the word "ABACUS"}

The symbol **|** or **:** denotes "such that".

Representation of Sets

Example: Which of the following qualify as sets ? Give reason.
(i) all persons living in India
(ii) A team of 6 best batsmen and 5 best bowlers of the world.

Solution: (i) It is a set, since the set criteria is well defined.
(ii) It is not a set, since there is no universally agreed criteria for deciding who are the 'best' batsmen and bowlers.

Example: Write the following sets in roster form:
(i) A = {x : x is a two digit natural number, sum of whose digits is 9}
(ii) $B = \{x : x \in \mathbf{Z}, x^3 < 100\}$
(iii) C = Set of alphabets in the word ALGEBRA.

Solution: (i) A = {90, 18, 81, 27, 72, 36, 63, 45, 54}
(ii) B = {…..,-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4}
(iii) C = {A, L, G, E, B, R}

Representation of Sets

Example: Write the set of all even natural numbers in set builder form.

Solution: $\{x : x = 2n, n \in \mathbf{N}\}$

Example: **Write the set A =** $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$ **in set builder form.**

Solution: $\mathbf{A} = \left\{\mathbf{x : x =} \frac{\mathbf{n}}{\mathbf{n+1}}, \mathbf{n} \in \mathbf{N}, \mathbf{n} \leq \mathbf{5}\right\}$

Example: How will you represent the set of all Irrational numbers ?

Solution: $\{x : x \in \mathbf{R}, x \notin \mathbf{Q}\}$

Types of Sets

➢ Empty Set

Empty or null or void set is one which has no element in it. It is denoted by ϕ.

Example: $\{x : x^2 - 3 = 0$ and x is a rational number$\} = \phi$

Example: $\{x : x < 2 \text{ and } x > 4\} = \phi$

Example: {x : x is a point common to two parallel lines} = ϕ

➢ Singleton Set

A set containing only a single element is called **Singleton set**.

Example: {a : a is an even prime number} = {2}, is a singleton set.

Types of Sets

➢ Finite and Infinite Set

A set which consists of a finite number of elements is called **Finite set,** otherwise it is called **Infinite set**.

Example: The set of positive integers less than 50 is a finite set.

Example: The set of integers less than 50 is an infinite set.

Example: The sets **N**, **Z**, **R** are infinite

Example: ϕ is a finite set (since 0 is finite).

Note: Since it is not possible to list out all elements of an infinite set, in Roster form such sets are sometimes indicated by writing a few elements to indicate structure of the set followed / preceded by dots. E.g. **Z** = {… -3,-2,-1,0,1,2,3 …}

❖ The number of elements in a set is called its **cardinal number** or its **order**. It is denoted by n(A) for any set A.

Types of Sets

➢ Equivalent and Equal Sets

Two finite sets A an B are **equivalent** if their cardinal numbers are same i.e., n(A)= n(B).

Sets A and B are said to be **equal** if they have exactly the same elements i.e., every element of A is a member of B and vice versa. It is denoted as A = B.

Example: If A = {3, 4, 7, 10} and B = {u, v, x, y}, then A and B are equivalent since n(A) = n(B) = 4, but A $\neq$ B since elements of each set are different.

Types of Sets

Example: Check whether the following sets are null, singleton or neither.
(i) C = {c : c is a circle passing through 3 given non-collinear points}
(ii) X = {x : $x < 2$ and $x \geq 2$}

Solution: (i) C is a singleton set. (ii) X is a null set.

Example: State whether the following sets are finite or infinite.
(i) A = Set of lines parallel to y axis.
(ii) X = Set of solutions of a Polynomial equation of degree n.

Solution: (i) A is an infinite set. (ii) X is a finite set.

Example: Check whether the following sets are equal.
A = {3, -2/3}, B = {x : x is a solution of $3x^2 + 7x - 6 = 0$}

Solution: On solving, we get B = {-3, 2/3}. $\therefore$ $A \neq B$.

Thanks…