(3) **Combination of drops :** Suppose we have n identical drops each having Radius r, Capacitance c, Charge q, Potential V and Energy u. If these drops are combined to form a big drop of Radius R, Capacitance C, Charge Q, Potential V and Energy U then

(i) Charge on big drop $Q = nq$

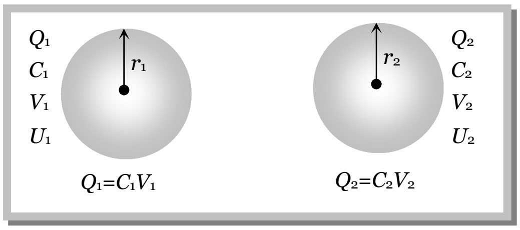
(ii) Radius of big drop $R = n^{1/3} r$

(iii) Capacitance of big drop $C = n^{1/3} c$

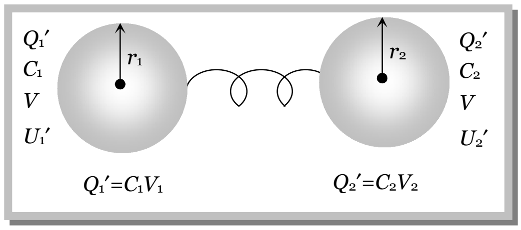
(iv) Potential of big drop $V = n^{2/3} V$ (*Problem based on this formula frequently asked in PET/PMT exams*)

(v) Energy of big drop $U = n^{5/3} u$

(4) **Redistribution of charge when two separately charged conductor are connected by a metallic wire :**



Before connection



After connection

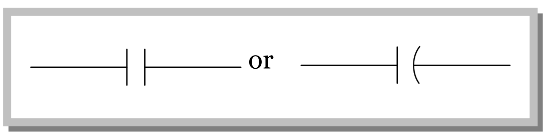
(i) **New charge :** $Q_2' = Q\left[\dfrac{r_2}{r_1 + r_2}\right]$ and $Q_1' = Q\left[\dfrac{r_1}{r_1 + r_2}\right]$

(ii) **Common potential :** $(V) = \dfrac{\text{Total charge}}{\text{Total capacity}}$; $\Rightarrow$ $V = \dfrac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

(iii) **Energy loss :** $\Delta U = U_f - U_i = \dfrac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Capacitor

A capacitor is a device that stores electric energy. It is also named condenser. The symbol of capacitor is shown in figure.

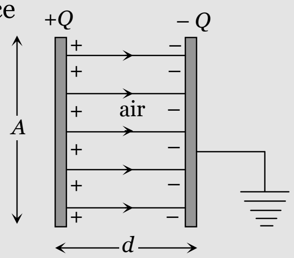
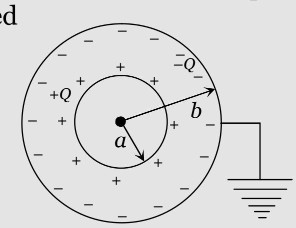
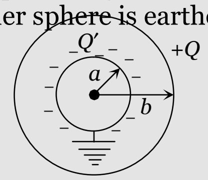
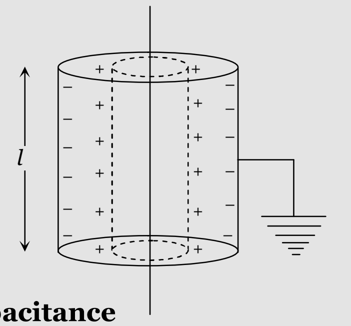


The capacitance of a capacitor is defined as the magnitude of the charge Q on the positive plate divided by the magnitude of the potential difference V between the plates i.e., $C = \dfrac{Q}{V}$

(1) **Energy stored :** When a capacitor is charged by a voltage source (say battery) it stores the electric energy. If C = Capacitance of capacitor; Q = Charge on capacitor and V = Potential difference across capacitor then energy stored in capacitor $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV = \dfrac{Q^2}{2C}$

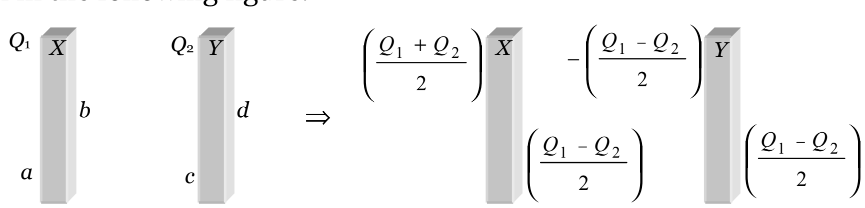
Note : ❑ In charging capacitor by battery half the energy supplied is stored in the capacitor and remaining half energy (1/2 QV) is lost in the form of heat.

(2) **Types of capacitors :** Capacitors are of mainly three types as described in given table

Parallel Plate Capacitor	**Spherical Capacitor**	**Cylindrical Capacitor**
It consists of two parallel metallic plates (may be circular, rectangular, square) separated by a small distance  A = Effective overlaping area of each plate d = *Separation between the plates* Q = Magnitude of charge on the inner side of each plate σ = Surface density of charge of each plate $\left(=\frac{Q}{A}\right)$ V = Potential difference across the plates E = Electric field between the plates $\left(=\frac{\sigma}{\varepsilon_0}\right)$ **Capacitance –** $C=\frac{\varepsilon_0 A}{d}$ in C.G.S. – $C=\frac{A}{4\pi d}$ If a dielectric medium of dielectric constant K is filled completely between the plates then capacitance increases by K times $C'=KC$	It consists of two concentric conducting spheres of radii a and b (a < b). Inner sphere is given charge +Q, while outer sphere is earthed  **Capacitance** $C=4\pi\varepsilon_0 \cdot \frac{ab}{b-a}$ in C.G.S. $C=\frac{ab}{b-a}$. In the presence of dielectric medium (dielectric constant K) between the spheres $C'=4\pi\varepsilon_0 K\frac{ab}{b-a}$ **Special Case** : If outer sphere is given a charge +Q while inner sphere is earthed  Induced charge on the inner sphere $Q'=-\frac{a}{b}.Q$, $C'=4\pi\varepsilon_0 \cdot \frac{b^2}{b-a}$ This arrangement is not a capacitor. But it's capacitance is equivalent to the sum of capacitance of spherical capacitor and spherical conductor i.e. $4\pi\varepsilon_0 .\frac{b^2}{b-a}=4\pi\varepsilon_0\frac{ab}{b-a}+4\pi\varepsilon_0 b$	It consists of two concentric cylinders of radii a and b (a < b), inner cylinder is given charge +Q while outer cylinder is earthed. Common length of the cylinders is l then  **Capacitance** $C=\frac{2\pi\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)}$ In the presence of dielectric medium (dielectric constant K) capacitance increases by K times and $C'=\frac{2\pi\varepsilon_0 Kl}{\log_e\left(\frac{b}{a}\right)}$

Note : ❑ The intensity of electric field between the plates of a parallel plate capacitor does not depends upon the distance between them.

❑ Two large conducting plates X and Y kept close to each other. The plate X is given a charge Q_1 while plate Y is given a charge Q_2 $(Q_1>Q_2)$, the distribution of charge on the four faces a, b, c, d will be as shown in the following figure.



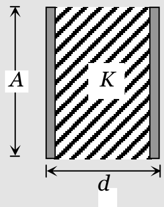
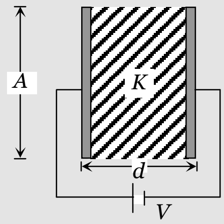
- Force and energy density between the plates of a parallel plate capacitor are respectively

$$|F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}\text{; Energy density} = \frac{1}{2}\varepsilon_0 E^2.$$

- 50% of the energy drawn from the battery stored in the capacitor.

Variation of *Q*, *C*, *V*, *E* and *U* of Parallel Plate Capacitor

(1) **When dielectric is completely filled between the plates**

Quantity	**Battery is Removed**	**Battery Remains connected**
		
Capacity	$C' = KC$	$C' = KC$
Charge	$Q' = Q$ (Charge is conserved)	$Q' = KQ$
Potential	$V' = V/K$	$V' = V$ (Since Battery maintains the potential difference)
Intensity	$E' = E/K$	$E' = E$
Energy	$U' = U/K$	$U' = U/K$

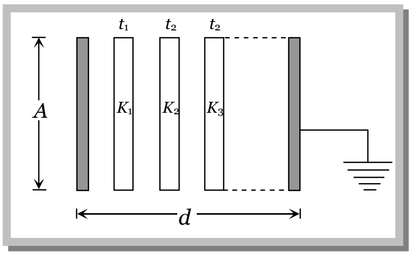
(2) **When separation between the plates is changing**

Quantity	**Battery is removed**		**Battery remains connected**	
	'*d*' is increased	**'*d*' is decreased**	**'*d*' is increased**	**'*d*' is decreased**
Capacity	Decreases	Increases	Decreases	Increases
Charge	Remains conserved	Remains conserved	Decreases	Increases
Potential	Increases	Decreases	Maintain constant by battery	Remains constant
Intensity	Remains unchanged	Remains unchanged	Decreases	Increases
Energy	Increases	Decreases	Decreases	Increases

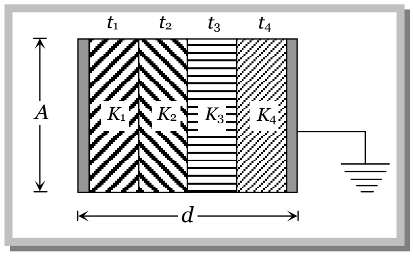
Other Important Cases of Parallel Plate Capacitor

(1) If a dielectric slab of thickness $t(t < d)$ is inserted between the plates, then $C' = \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{k}}$. To maintain the capacitance and potential difference of capacitor as before separation between the plates has to be increased. Suppose separation is increased by d' so in this case $k = \dfrac{t}{t - d'}$.

(2) If so many dielectric slabs are inserted between the plates as shown then

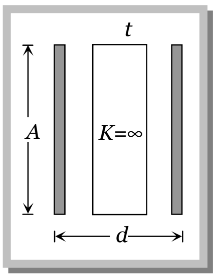


$$C' = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + ...) + \left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + ...\right)}$$

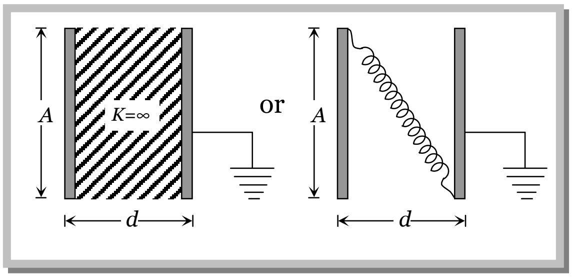


$$C' = \frac{\varepsilon_0 A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + \frac{t_4}{k_4}\right)}$$

(3) When a metallic slab of thickness $t(t \le d)$ is partially filled the gap between the plates.

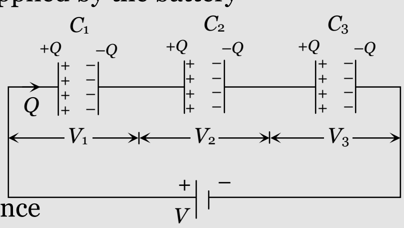
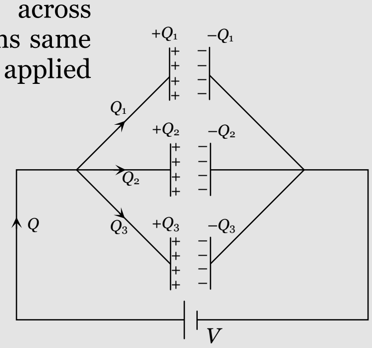


$$C' = \frac{\varepsilon_0 A}{(d - t)}$$



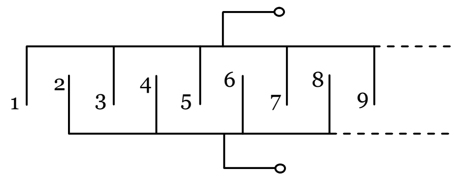
$$C' = \infty$$

Grouping of Capacitors

Series grouping	Parallel grouping
(1) Charge on each capacitor remains same and equals to the main charge supplied by the battery  $V = V_1 + V_2 + V_3$	(1) Potential difference across each capacitor remains same and equal to the applied potential difference  $Q = Q_1 + Q_2 + Q_3$
(2) Equivalent capacitance $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$ or $C_{eq} = (C_1^{-1} + C_2^{-1} + C_3^{-1})^{-1}$	(2) $C_{eq} = C_1 + C_2 + C_3$
(3) In series combination potential difference and energy distribution in the reverse ratio of capacitance i.e., $V \propto \dfrac{1}{C}$ and $U \propto \dfrac{1}{C}$.	(3) In parallel combination charge and energy distributes in the ratio of capacitance *i.e.* $Q \propto C$ and $U \propto C$
(4) If two capacitors having capacitances C_1 and C_2 are connected in series then $\boldsymbol{C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}} = \dfrac{\text{Multiplication}}{\text{Addition}}$	(4) If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel then $\boldsymbol{C_{eq} = C_1 + C_2}$

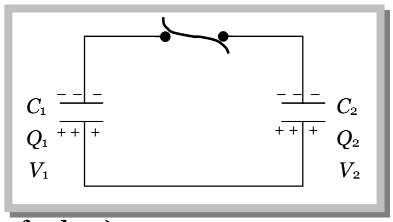
$V_1 = \left(\frac{C_1}{C_1 + C_2}\right).V$ and $V_2 = \left(\frac{C_2}{C_1 + C_2}\right).V$	$Q_1 = \left(\frac{C_1}{C_1 + C_2}\right).Q$ and $Q_2 = \left(\frac{C_2}{C_1 + C_2}\right).Q$
(5) If n identical capacitors each having capacitances C are connected in series with supply voltage V then Equivalent capacitance $C_{eq} = \frac{C}{n}$ and Potential difference across each capacitor $V' = \frac{V}{n}$.	(5) If n identical capacitors are connected in parallel Equivalent capacitance $C_{eq} = nC$ and Charge on each capacitor $Q' = \frac{Q}{n}$

Note : ❑ If n identical plates are arranged such that even no. of plates are connected together and odd number of plates are connected together, then $(n - 1)$ capacitors will be formed and they will be in parallel grouping. Equivalent capacitance $C' = (n-1)C$ where $C = \frac{\varepsilon_0 A}{d}$



Redistribution of Charge Among Two Capacitors

When a charged capacitor is connected across an uncharged capacitor, then redistribution of charge occur to equalize the potential difference across each capacitor. Some energy is also wasted in the form of heat.



Suppose we have two charged capacitors C_1 and C_2 after disconnecting these two from their respective batteries. These two capacitors are connected to each other as shown below (positive plate of one capacitor is connected to positive plate of other while negative plate of one is connected to negative plate of other)

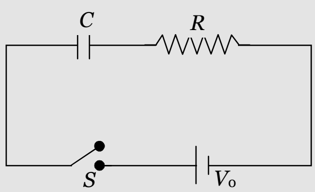
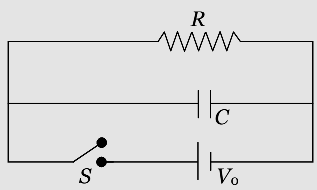
Charge on capacitors redistributed and new charge on them will be $Q_1' = Q\left(\frac{C_1}{C_1 + C_2}\right)$, $Q_2' = Q\left(\frac{C_2}{C_1 + C_2}\right)$

The common potential $V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$ and loss of energy $\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Note : ❑ Two capacitors of capacitances C_1 and C_2 are charged to potential of V_1 and V_2 respectively. After disconnecting from batteries they are again connected to each other with reverse polarity *i.e.*, positive plate of a capacitor connected to negative plate of other. So common potential $V = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1 V_1 - C_2 V_2}{C_1 + C_2}$

Circuit With Resistors and Capacitors

(1) A resistor may be connected either in series or in parallel with the capacitor as shown below

Series RC Circuit	Parallel RC Circuit
	
In this combination capacitor takes longer time to	Resistor has no effect on the charging of capacitor.

charge.	
The charging current is maximum in the beginning; it decreases with time and becomes zero after a long time.	Resistor provides an alternative path for the electric current.

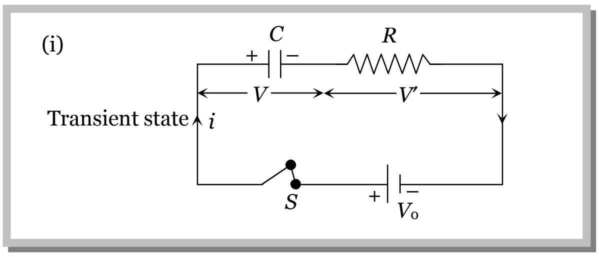
(2) **Three states of *RC* circuits**

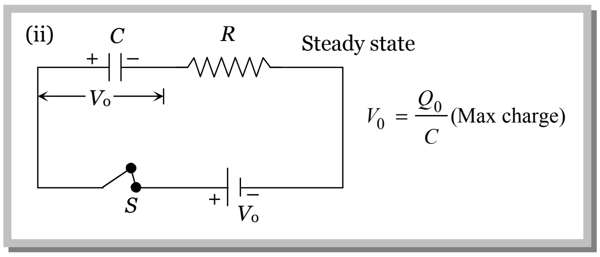
(i) Initial state : i.e., just after closing the switch or just after opening the switch.

(ii) Transient state : or instantaneous state i.e., any time after closing or opening the switch.

(iii) Steady state : i.e., a long time after closing or opening the switch. In the steady state condition, the capacitor is charged or discharged.

(3) **Charging and discharging of capacitor in series RC circuit** : As shown in the following figure (i) when switch *S* is closed, capacitor start charging. In this transient state potential difference appears across capacitor as well as resistor. When capacitor gets fully charged the entire potential difference appeared across the capacitor and nothing is left for the resistor. [shown in figure (ii)]



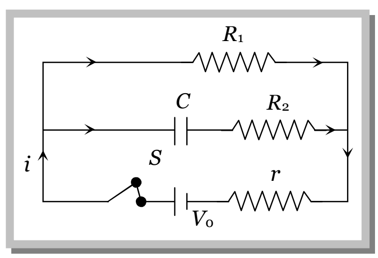


(i) **Charging :** In transient state of charging charge on the capacitor at any instant $Q = Q_0\left(1 - e^{\frac{-t}{RC}}\right)$ and potential difference across the capacitor at any instant $V = V_0\left(1 - e^{\frac{-t}{RC}}\right)$

(ii) **Discharging :** After the completion of charging, if battery is removed capacitor starts discharging. In transient state charge on the capacitor at any instant $Q = Q_0 e^{-t/RC}$ and potential difference cross the capacitor at any instant $V = V_0 e^{-t/CR}$.

(iii) **Time constant (τ) :** The quantity *RC* is called the time constant as it has the dimension of time. In charging If $t = \tau = RC$, $Q = Q_0(1 - e^{-1}) = 0.63\, Q_0$ = 63% of Q_0 ($\frac{1}{e} = 0.37$) **or** In discharging it is defined as the time during which charge on a capacitor falls to 0.37 times (37%) of the initial charge on the capacitor.

(iv) **Mixed *RC* circuit :** In the following circuit when capacitor gets fully charged, potential difference across capacitor will be equal to potential difference across R_1 *i.e.* $\frac{V_0}{(R_1 + r)}$ so in steady state charge on capacitor is $Q = \frac{CV_0 R_1}{R_1 + r}$



Network Solving

To solve capacitive network for equivalent capacitance following guidelines should be followed.

Guideline 1. Identify the two points between which the equivalent capacitance is to be calculated.

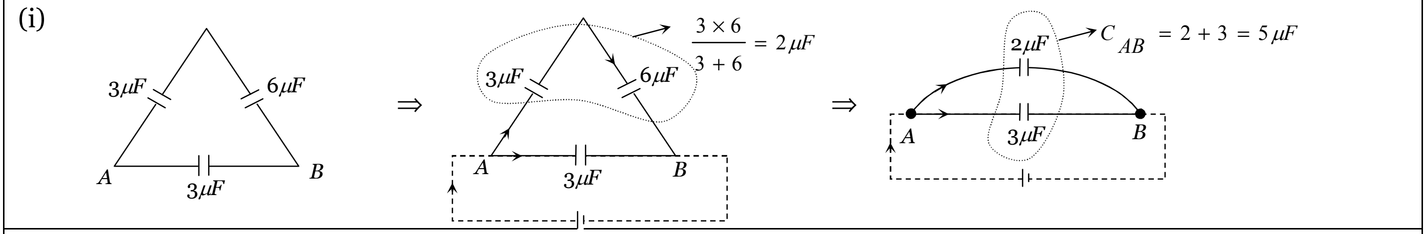
Guideline 2. Connect (Imagine) a battery between these points.

Guideline 3. Solve the network from the point (reference point) which is farthest from the points between which we have to calculate the equivalent capacitance. (The point is likely to be not a node)

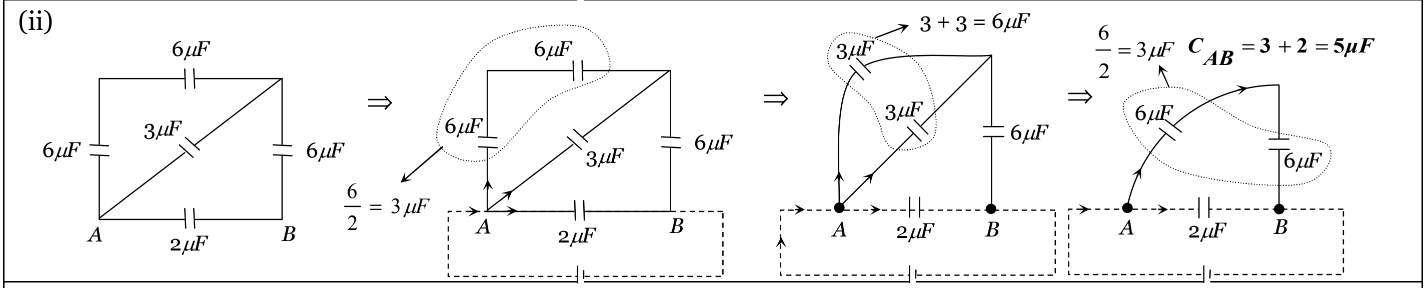
(1) **Simple circuits**

Suppose equivalent capacitance is to be determined in the following networks between points A and B

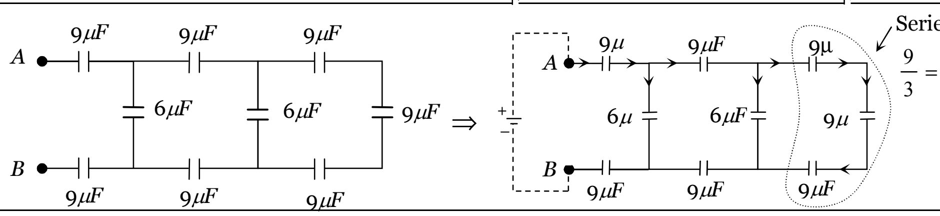
(i)



(ii)



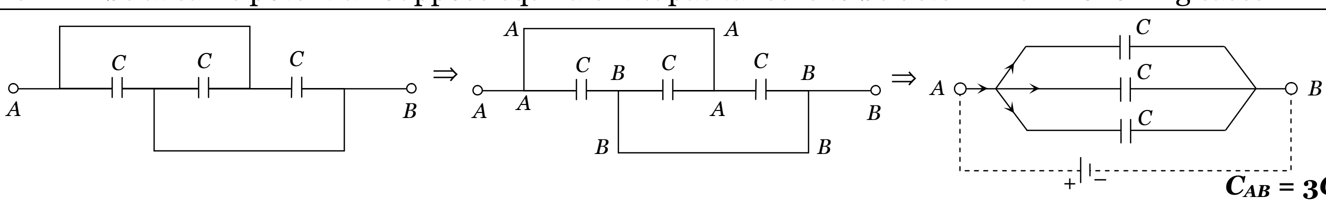
(iii)



⇒ By similar process C_{AB} = **3μF**

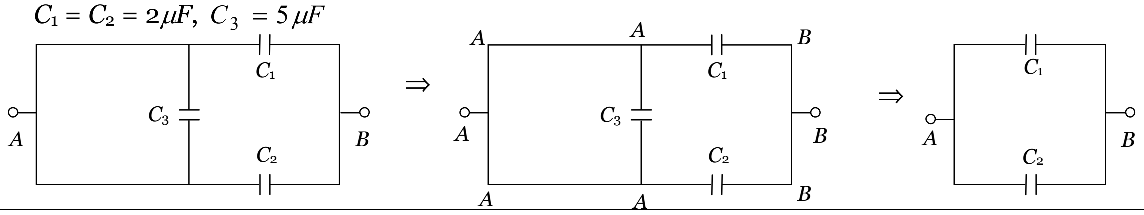
(2) **Circuits with extra wire :** If there is no capacitor in any branch of a network then every point of this branch will be at same potential. Suppose equivalent capacitance is to be determine in following cases

(i)



C_{AB} = **3C**

(ii) $C_1 = C_2 = 2\mu F,\ C_3 = 5\mu F$



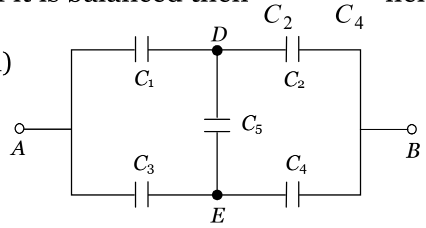
No p.d. across C_3 so it is removed

C_{AB} = **4μF**

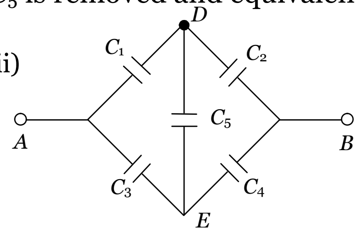
(3) **Wheatstone bride based circuit :** If in a network five capacitors are arranged as shown in following figure, the network is called wheatstone bridge.

If it is balanced then $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ hence C_5 is removed and equivalent capacitance between A and B

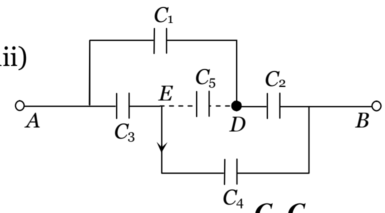
(i)



(ii)

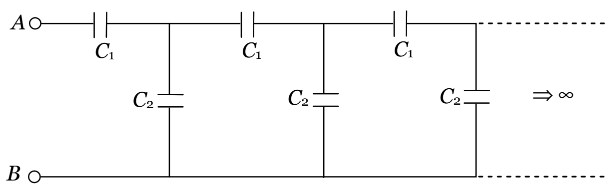


(iii)



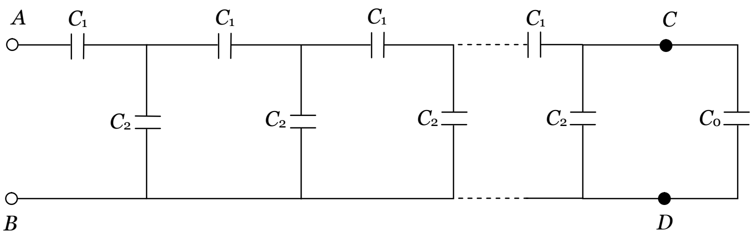
$$C_{AB} = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$$

(4) **Infinite chain of Capacitors :** In the following figure equivalent capacitance between A and B



$$C_{AB} = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$$

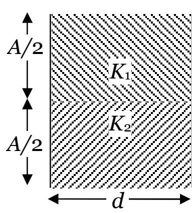
The value of C_0 in the circuit shown for which the net effective capacitance between A and B be independent of the number of sections in the chain



$$C_0 = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$$

(5) **Advance case of compound dielectrics :** If several dielectric medium filled between the plates of a parallel plate capacitor in different ways as shown.

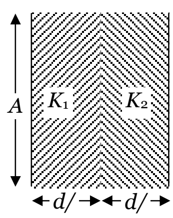
(i)



$$C_{eq} = \left(\frac{K_1 + K_2}{2}\right)\frac{\varepsilon_0 A}{d}$$

$$K_{eq} = \frac{K_1 + K_2}{2}$$

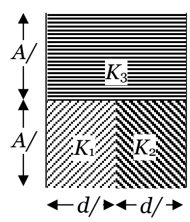
(ii)



$$C_{eq} = \left(\frac{2K_1K_2}{K_1 + K_2}\right).\frac{\varepsilon_0 A}{d}$$

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

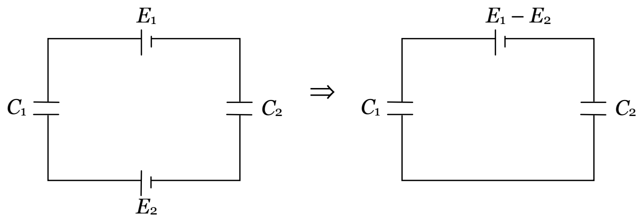
(iii)



$$C_{eq} = \left(\frac{K_1K_2}{K_1 + K_2} + \frac{K_3}{2}\right)\frac{\varepsilon_0 A}{d}$$

$$K_{eq} = \left(\frac{K_3}{2} + \frac{K_1K_2}{K_1 + K_2}\right)$$

(6) **Network with more than one cell**



Potential difference across C_1 is $\left(\frac{C_2}{C_1 + C_2}\right)(E_1 - E_2)$ and potential difference across C_2 is $\left(\frac{C_1}{C_1 + C_2}\right)(E_1 - E_2)$