ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

Contents

5. ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

5.1 ELECTROMAGNETIC INDUCTION

5.1.1 Introduction

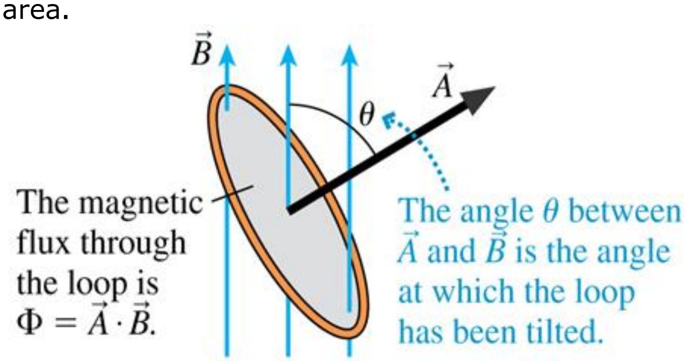
In 1831, Michael Faraday discovered the effect, called electromagnetic induction, just converse to the magnetic effect of current.
Electromagnetic induction is the phenomenon of production of electric current or e.m.f. in a coil, when magnetic flux linked with the coil is changed. The current and the e.m.f. so produced are called induced current and induced e.m.f.

5.1.2 Magnetic flux

The magnetic flux linked with a surface, when held in a magnetic field is defined as the number of magnetic field lines crossing the surface normally and is measured as the product of component of the magnetic field normal to the surface and the surface area.
It is scalar quantity and is denoted by Φ.

$$\Phi = BA\cos\theta$$

Where B is magnetic field, A is surface area and θ is the angle between magnetic field and normal to the surface area.



Case 1: When magnetic field is perpendicular to the surface area i.e. when θ = 0°

$$\Phi = BA$$

Magnetic flux linked with the surface is maximum.

Case 2: When magnetic field is parallel to the surface area i.e. when θ = 90°

$$\Phi = 0$$

Unit of magnetic flux is weber (Wb)

5.1.3 Faraday's laws of electromagnetic induction

First law: Whenever magnetic flux linked with a circuit changes, induced e.m.f. is produced.

Second law: The induced e.m.f. lasts as long as the change in the magnetic flux continues.

Third law: The magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux linked with the circuit.

$$\text{i.e. } e = -\frac{d\varphi}{dt}$$

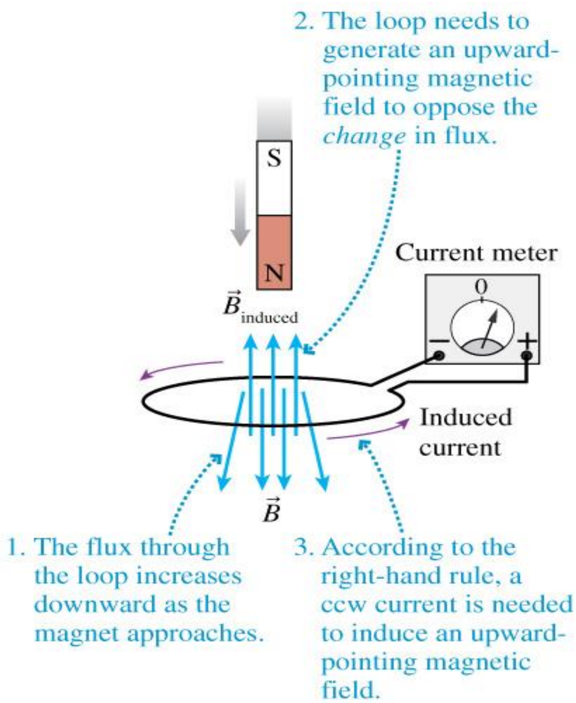
Negative sign indicates that the induced e.m.f. e has got opposing nature.

5.1.4 Induced EMF and Current

The changing magnetic field causes an induced current. Since a source of emf (voltage) is needed to produce a current (remember Ohm's Law), the moving magnet acts like a source of emf. So we would say the moving magnet induces an emf in the coil, producing an induced current.

5.1.5 Lenz's law

Lenz's law states that the induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.



There is an induced current in a closed conducting loop only if the magnetic flux is changing (either **B, A** or θ). The direction of the induced current is such that the *induced* magnetic field opposes the change.

Using Lenz Law

1. **The direction of the external magnetic field can be determined.**
2. **Determine how the flux is changing.** Is it increasing, decreasing, or staying the same?
3. **Determine the direction of an induced magnetic field that will oppose the change in the flux.** Increasing: induced magnetic field points opposite the external magnetic field. Decreasing: induced magnetic field points in the same direction as the external magnetic field. Constant: no induced magnetic field.

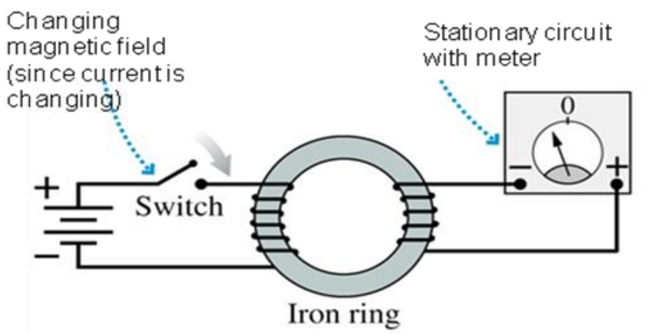
4. **Determine the direction of the induced current.** Using the right-hand rule.

5.1.6 Lenz's Law and Conservation of Energy

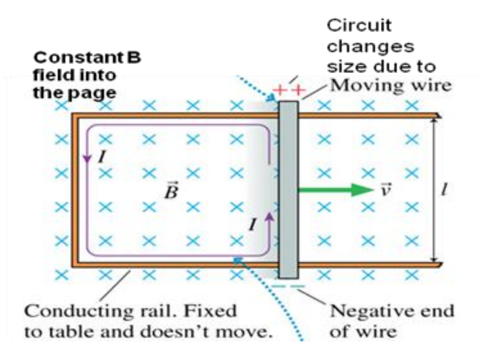
Lenz's law is a consequence of the law of conservation of energy. According to the law of conservation of energy the total amount of energy in the universe must remain constant. Energy can be neither created nor destroyed. Hence it is impossible to get free energy from nothing.
Push a bar magnet through a coil of wire. The moving magnet induces an electric current in the wire, which in turn induces its own magnetic field. According to Lenz's law, the induced magnetic field opposes the cause, which is the moving magnet. Hence the induced magnetic field is in a direction to try to stop the moving magnet. If this were not the case, the induced magnetic field would increase the magnet's velocity and thereby increase its kinetic energy. There is no source for this energy. So if the induced magnetic field helped rather than opposed its cause, conservation of energy would be violated.

5.1.7 Motional emf

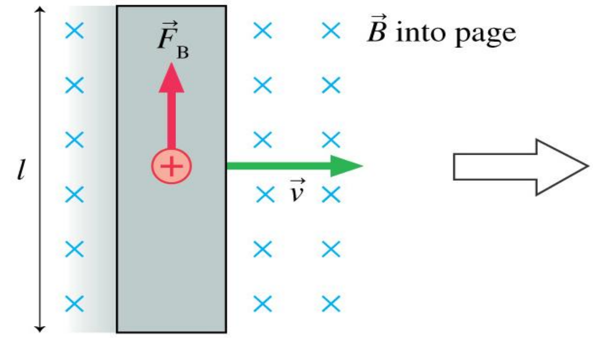
An induced current in a circuit can be created 2 ways:
1. By changing the strength of the magnetic field through a stationary circuit.



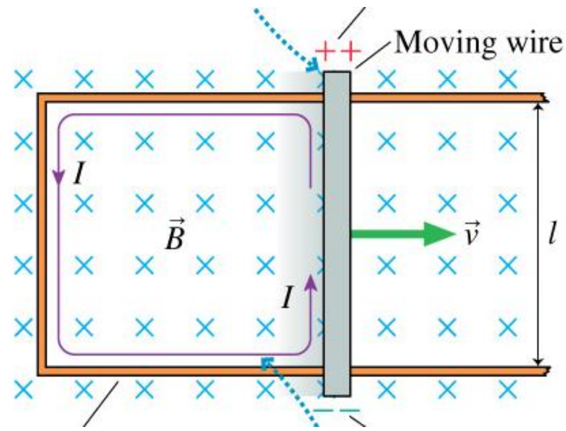
2. By changing the size or orientation of the circuit in a stationary magnetic field. We shall look at this second method first. This is motional emf.



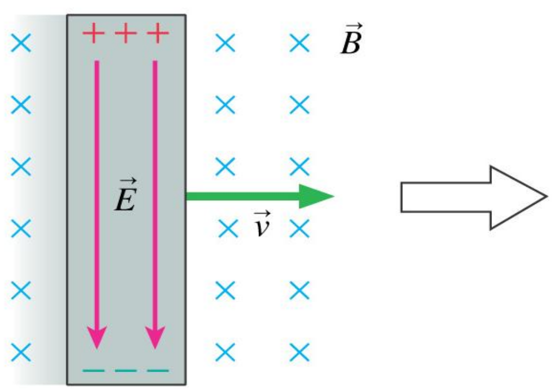
The external magnetic field causes a magnetic force on positive and negative charges moving to the right. The electron holes move *up* and the electrons move *down*.



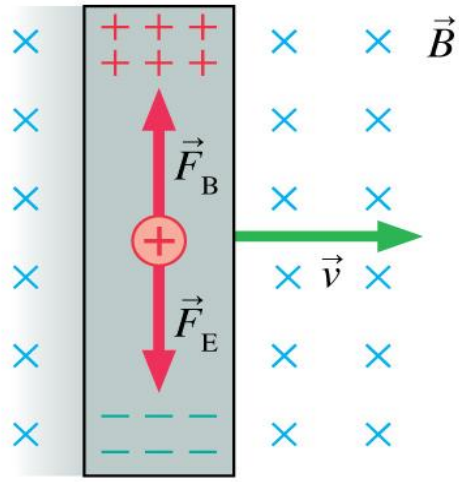
Since the rest of the loop isn't moving, there is no magnetic force on it, so the electron holes will flow along the wire to get back to the more negative side.



Another result of the charge separation is an Electric field.
This Electric field also causes a force, which is in the opposite direction as the force due to the B field.



Electron holes continue to move *up*, but only until F_E *down* equals F_B *up*, at which charge separation ceases. As long as the wire keeps moving, there will be a charge separation. The magnetic force is doing work to maintain that charge separation.

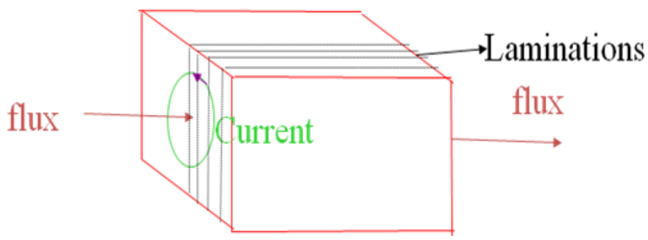


The expression for motional emf is

$$e = -Blv$$

5.1.8 Eddy currents

The currents induced in the body of a conductor, when the magnetic flux linked with the conductor changes, are called eddy currents (or Focault's currents).



When current is induced in a conductor such as the square piece of metal shown above, the induced current often flows in small circles that are strongest at the surface and penetrate a short distance into the material. These current flow patterns are thought to resemble eddies in a stream, which are the tornado looking swirls of the water that we sometimes see. Because of this presumed resemblance, the electrical currents were named **eddy currents**.

5.1.9 Self Induction

Inductance is the property in an electrical circuit where a change in the electric current through that circuit induces an electromotive force (EMF) that opposes the change in current
Self induction is the property of a coil by the virtue of which it opposes any change in the strength of current flowing through it by inducing an e.m.f. in itself. For this reason self-inductance is also called the inertia of electricity.
Inductors are coils which can oppose the changes of current in a circuit. They are used for reducing current in AC circuits without any loss of electrical energy. For these purposes resistors can also be used. But while using resistors electrical energy is wasted in the form of heat.

5.1.10 Coefficient of self induction

The coefficient of self induction or simply self inductance (L) of a coil is numerically equal to the magnetic flux linked with it when unit current flows through it.

i.e. $\Phi = LI$

The self inductance of a coil is also numerically equal to the induced emf produced in the coil, when the rate of change of current in the coil is unity.

i.e. $e = -L(dI/dt)$

SI unit of self inductance is henry (H).

Example: What e.m.f. will be induced in a 10 H inductor in which current changes from 10A to 7A in $9\text{x}10^{-2}$ s?

Solution: L= 10H, I_1= 10A, I_2= 7A, dt= $9\text{x}10^{-2}$s

As $e = -L\, dI/dt$,

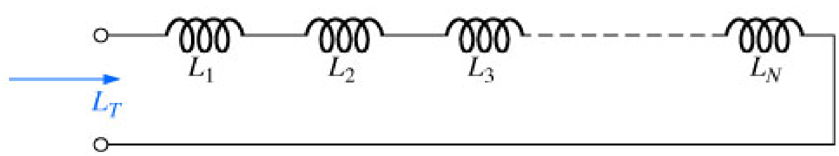
$= -L (I_2 - I_1)/dt$

$= -10 (7-10)/ 9\text{x}10^{-2}$

$= 333.3$ Volt

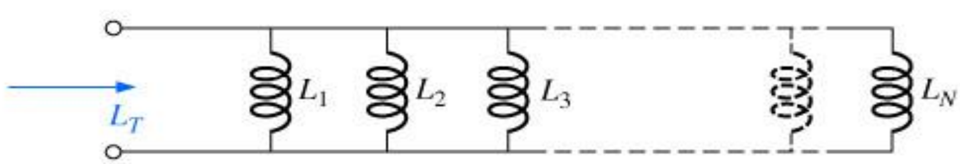
5.1.11 Grouping of inductors

1) Inductors in series: Inductors, like resistors and capacitors, can be placed in series. Increasing levels of inductance can be obtained by placing inductors in series.



$$L_T = L_1 + L_2 + L_3 + \cdots + L_N$$

2) Inductors in parallel: Decreasing levels of inductance can be obtained by placing inductors in parallel.



$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

5.1.12 Energy stored in an inductor

Suppose that an inductor of inductance L is connected to a variable DC voltage supply. The supply is adjusted so as to increase the current i flowing through the inductor from zero to some final value I. As the current through the inductor is ramped up, an emf

$$\mathcal{E} = -L\,di/dt$$

is generated, which acts to oppose the increase in the current. Clearly, work must be done against this emf by the voltage source in order to establish the current in the inductor. The work done by the voltage source during a time interval dt is

$$dW = P\,dt = -\mathcal{E}\,i\,dt = i\,L\,\frac{di}{dt}\,dt = L\,i\,di.$$

Here,

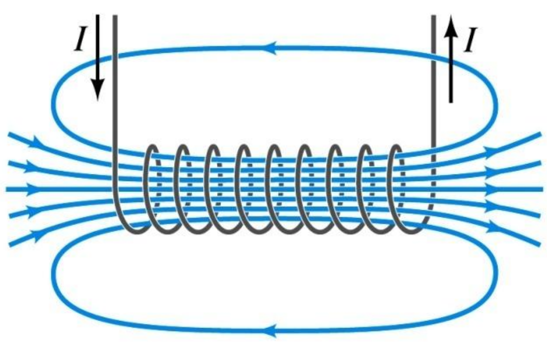
$$P = -\mathcal{E}\,i$$

is the instantaneous rate at which the voltage source performs work. To find the total work done W in establishing the final current I in the inductor, we must integrate the above expression. Thus,

$$W = L\int_0^I i\,di,$$

$$W = \frac{1}{2}\,L\,I^2.$$

5.1.13 Self inductance of a long solenoid



The magnetic flux through each turn is

$$\Phi_B = BA = \left(\mu_o \frac{N}{l} I\right)A$$

Where N is the total number of turns, l is the length of the solenoid, I is the current through the solenoid and A is the cross sectional area of the solenoid.

Therefore, the inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_o N^2 A}{l}$$

This shows that *L* depends on the geometry of the object.

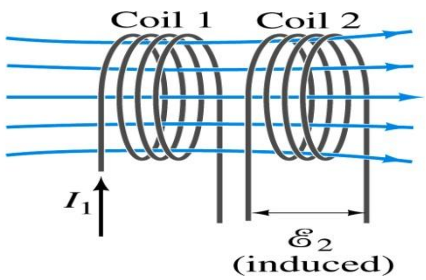
5.1.14 Energy stored in a solenoid

When a current is passed through a solenoid of length l and cross sectional area A, the energy stored inside it is in the form of magnetic field. If the current builds up a magnetic field of induction B, then the energy stored in the solenoid is given by

$$W = \frac{1}{2\mu_0} B^2 A l$$

5.1.15 Mutual induction

Mutual induction is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.



5.1.16 Coefficient of mutual induction

The coefficient of mutual induction or simply mutual inductance (M) of the two coils is numerically equal to the magnetic flux (Φ) linked with one coil, when a unit current flows through the neighboring coil.

i.e. $\Phi = MI$

The mutual inductance of two coils is also numerically equal to the induced emf produced in one coil, when rate of change of current is unity in the other coil.

i.e. $e = -M(dI/dt)$

SI unit of mutual inductance is henry (H).

5.1.17 Mutual inductance of two long solenoids

Let S_1 solenoid air core has N_1 turns and another solenoid S_2 has N_2 turns is wound over the solenoid. l is equal to the length of each solenoid. Magnetic field B_1 at any point inside S_1 due to a current I_1 is,

$$B_1 = \mu_0 N_1 I_1 / l$$

Total magnetic flux linked with solenoid S_2,

$$\Phi_2 = B_1 \times A \times N_2$$

$$= (\mu_0 N_1 I_1 / l) \times A \times N_2$$

But, magnetic flux Φ_2 linked with S_2 due to current I_1 through S_1.

$$\text{So, } \Phi_2 \propto I_1$$

$$\Phi_2 = M I_1$$

$$M = (\mu_0 N_1/l)\, A x N_2$$

Example: What is the mutual inductance of a pair of coils if a current change of 6 A in one coil causes the flux in the second coil of 2000 turns to change by $12\text{x}10^{-4}$ Wb per turn?

Solution: I = 6A, N = 2000

$\Phi = 2000\text{x}12\text{x}10^{-4} = 2.4$ Wb

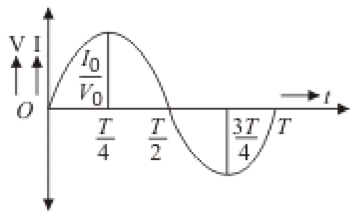
Φ= MI

M = Φ/I = 2.4/6 = 0.4

5.2 ALTERNATING CURRENTS

5.2.1 Alternating Current and Voltage

An alternating current is the current (or voltage) whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.



The common alternating current varying as sine function of the time is given by

$$I = I_0 \sin \omega t = I_0 \sin 2\pi v t = I_0 sin\frac{2\pi t}{T}$$

And $$V = V_0 sin 2\pi v t = V_0 sin\frac{2\pi t}{T}$$

Here, I_0 and V_0 are the maximum or peak values of current and voltage, ω the angular frequency, v the frequency and T the period of given AC.

In our domestic circuits the frequency of AC is 50 Hz. Thus, it changes its direction after every $\frac{1}{100} s$.

The average value of an AC for one complete cycle is zero.

Generally average or mean value of an AC means the average value of given AC over a half cycle

$$i.e, t = 0\ to\ \frac{T}{2}$$

$$\therefore \quad I_{av} = \frac{\int_0^{T/2} I\, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637\ I_0$$

and $$V_{av} = 0.637 V_0$$

The rms value of an AC is defined as

$$(I_{rms})^2 = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{I_0^2}{2} \Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}}$$

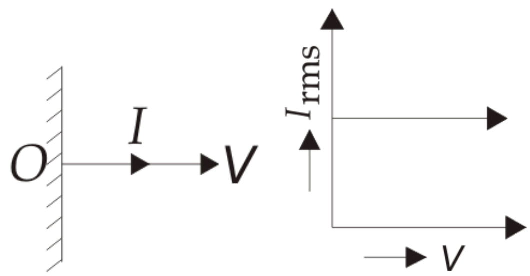
Similarly; $V_{rms} = \frac{V_0}{\sqrt{2}}$

5.2.2 AC Circuit Elements

5.2.2.1 Pure Resistive Circuit

Let an alternating voltage V = V_0sin ωt be applied across a pure resistance R. Then

Current $I = \frac{V}{R}$ or $I_{rms} = \frac{V_{rms}}{R}$

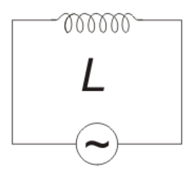


Current and voltage are in same phase, and current is given by I = I_0sin ωt.

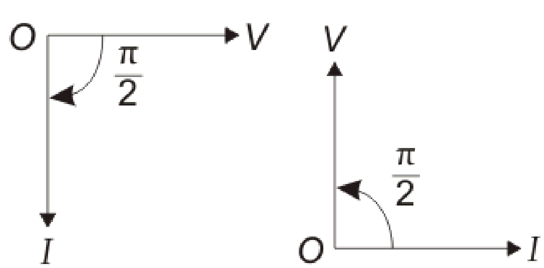
5.2.2.2 Pure inductive Circuit

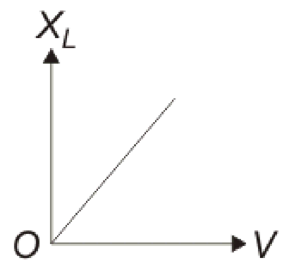
Let an alternating voltage V = V_0sin ωt be applied across a pure inductance I. Then average power $= V_{rms} I_{rms} \cos\frac{\pi}{2} = 0$

Such a current, for which average power as well as power factor is zero, is called the wattless current.



$V = V_0 \sin \omega t$





The inductance offers some opposition for the flow of AC. It is known as "inductive reactance" $X_L = 2\pi v L = L\omega$. Thus, a pure inductance does not oppose flow of DC (ω = 0) but oppose flow of AC.

Current flowing I = $\frac{V}{X_L}$

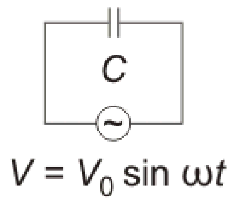
The current decreases with increase in frequency.

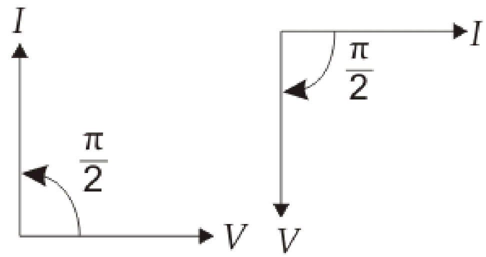
The current lags behind the voltage by $\frac{\pi}{2}$ (or voltage leads the current by $\frac{\pi}{2}$) and is thus gives by

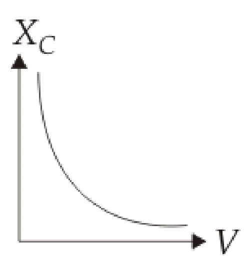
$$I = I_0 sin\left(\omega t - \frac{\pi}{2}\right)$$

5.2.2.3 Pure Capacitive Circuit

Let an alternating voltage V = V_0sin ωt be applied across a pure capacitance C. Then the capacitance offers some opposition for flow of current but allows AC to pass through it. The opposition offered is known capacitive reactance.







$$X_C = \frac{1}{C\omega}\Omega = \frac{1}{C.2\pi v}\Omega$$

Current flowing I = $\frac{V}{X_C}$

The current increases with increase in frequency

The current leads the voltage by $\frac{\pi}{2}$ (or voltage is behind the current by $\frac{\pi}{2}$) and is thus given by

$$I = I_0 sin\left(\omega t + \frac{\pi}{2}\right)$$

5.2.3 Reactance and Impedance

The opposition offered by a pure inductor or capacitance or both to the flow of AC through it is called reactance X. Its unit is ohm (Ω) and dimensional formula $[ML^2T^{-3}A^{-2}]$. Reactance is of two types:

(i) Inductive reactance $X_L = L\omega$ and

(ii) Capacitive reactance $X_C = \frac{1}{C\omega}$

Reciprocal of reactance to known as susceptance

Thus, $S = \frac{1}{X}$

Total opposition offered by an AC circuit to flow through circuit is called its impedance Z. Its unit is ohm and dimensional formula is $[ML^2T^{-3}A^{-2}]$. For a 1 A circuit

$$Z = \sqrt{X^2 + R^2} = \sqrt{(X_L - X_C)^2 + R^2}$$

Reciprocal of impedance is known as admittance. Thus, $Y=\frac{1}{Z}$. Its unit is siemen (S).

5.2.4 Power in an AC Circuit

Let a voltage $V = V_0 \sin \omega t$ be applied across an AC and consequently a current $I = I_0 \sin(\omega t - \phi)$ flows through circuit. Then

Instantaneous power = $VI = V_0 I_0 \sin \omega t \sin(\omega t - \phi)$ and its value varies with time.

Average power over a full cycle of AC

$$P_{av} = V_{rms}I_{rms}cos\phi = \frac{1}{2}V_0I_0cos\phi$$

The term $V_{rms}I_{rms}$ is known as apparent or virtual power but $V_{rms}I_{rms}cos\phi$ is called the true power.

The term cosϕ is known as power factor of given circuit. Thus,

$$cos\phi = \frac{R}{2} = power\ factor = \frac{true\ power}{apparent\ power}$$

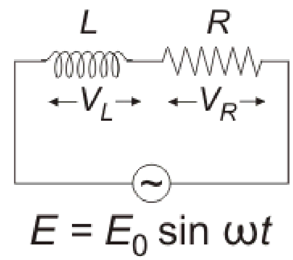
For a pure resistive circuit V and I are in phase Φ=0°, hence cosΦ = 1 and average power = $V_{rms}I_{rms}$

For a pure inductive or a pure capacitive circuit current and voltage differ in phase by $\frac{\pi}{2}\left(\phi = \frac{\pi}{2}\right)$ and average power = 0.

5.2.5 Series AC Circuits

5.2.5.1 Series LR Circuit

For $E=E_0 \sin \omega t$

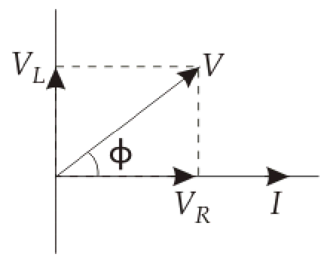


$$I = \frac{E_0}{Z} \sin(\omega t + \phi)$$

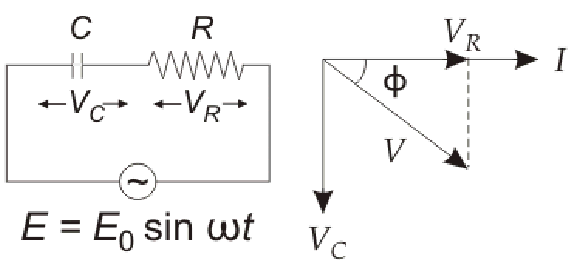
Where $Z=\sqrt{R^2 + (\omega L)^2}$ and $\tan\Phi = \frac{\omega L}{R}$

$$V=\sqrt{V_R^2 + V_L^2}$$

Current is lagging the voltage by ϕ.



5.2.5.2 Series R-C Circuit



For $E = E_0 \sin \omega t$,

$$I = \frac{E_0}{Z} \sin(\omega t - \phi)$$

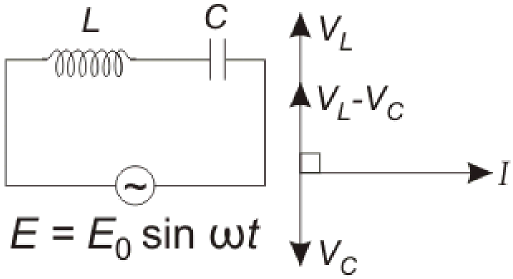
Where $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

And $\tan\phi = \frac{-1/\omega C}{R}$

Current leading the voltage by ϕ.

$$V^2 = V_R^2 - V_C^2$$

5.2.5.3 Series L-C Circuit

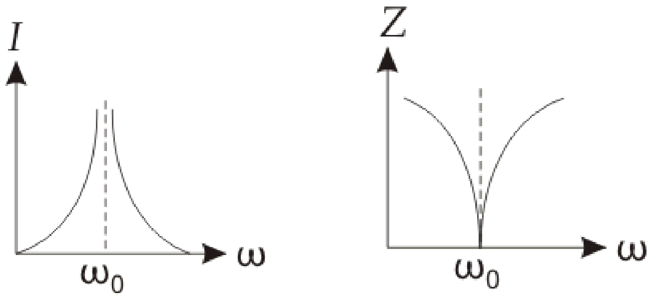


For E = E_0sin ωt, $I = \frac{E_0}{Z} \sin(\omega t - \phi)$

Where Z=$X_L - X_C and\, tan\phi = \frac{X_L - X_C}{R}$

For $X_L > X_C$, $\phi = \frac{\pi}{2} and\ for\ X_L < X_C, \phi = -\frac{\pi}{2}$

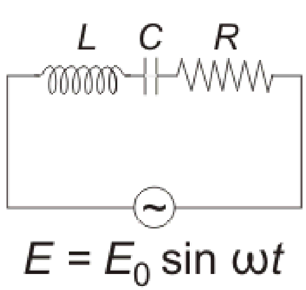
If X_L = X_C i.e, at $\omega = \frac{1}{\sqrt{LC}}$, Z = 0 and I_0 becomes infinity. This condition is termed as resonant condition and this frequency is termed as natural frequency of circuit.



Variation of I with ω in L-C circuit

Variation of Z with ω in L-C circuit

5.2.5.4 Series L-C-R Circuit

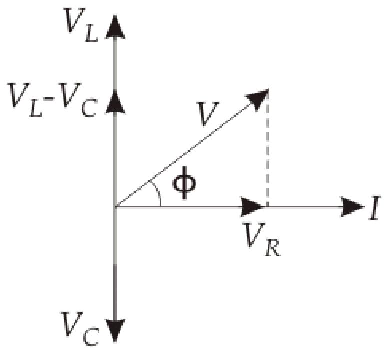


For E=E$_0$sin ωt, $I = \frac{E_0}{Z}\sin(\omega t - \phi)$

Where Z=$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

And $tan\phi = \frac{X}{R} = \frac{X_L - X_C}{R}$

For $X_L > X_C$ current lags voltage
$X_L < X_C$ current leads voltage
$X_L = X_C$ current and voltage are in phase
If $X_L = X_C \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ i.e, the natural frequency of the circuit is equal to applied frequency, then the circuit is said to be in resonance.



At resonance current in the circuit is maximum and impedance is minimum.
At resonance, $I_0 = \frac{E_0}{R}$ and

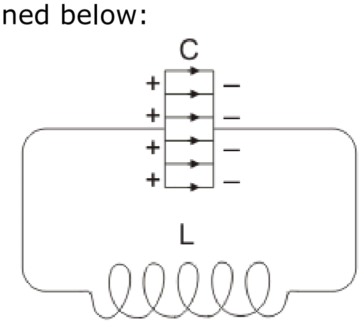
$$V_L = I_0X_L = V_C = I_0X_C$$

i.e, $V_L = \frac{\omega L}{R} \times E_0 = \frac{1}{\omega RC} \times E_0 = QE_0$

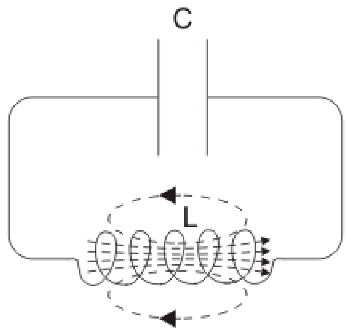
Where $Q = \frac{\omega L}{R} or \frac{1}{\omega RC}$ is termed as Quality factor circuit. It determines the sharpness of resonance. Higher the value of Q sharper is the resonance.

5.2.6 LC-Oscillations

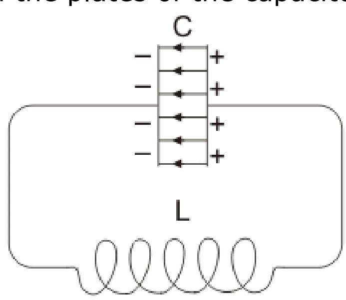
Consider a charged capacitor of capacitance C. The energy is stored in the capacitor in the form of electric field between its two plates. If the charged capacitor is connected to an inductor LC-oscillations are produced as explained below:



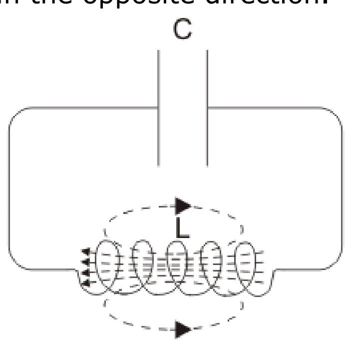
As soon as the capacitor is connected to inductor, it sends a current, thereby producing growing magnetic field inside the inductor. In turn, it produces induced e.m.f., which opposes the growth of current and hence the capacitor takes some finite time to discharge completely. When it discharges completely, the energy which was stored inside the capacitor in the form of electric field now appears in the form of magnetic field inside the inductor.



As soon as the discharge is complete, the current ceases to flow and the magnetic field lines linked with the inductor also start winding up. Owing to this, opposing induced e.m.f. is produced in the inductor. The opposing induced e.m.f. starts charging the capacitor but with opposite polarity. Now, the energy stored in the inductor in the form of magnetic field appears in the form of electric field between the plates of the capacitor.

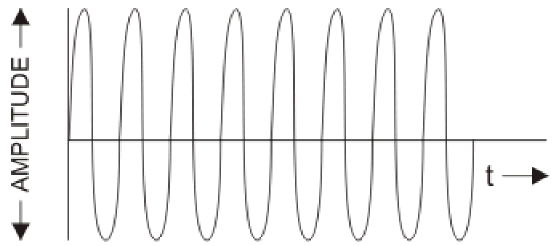


On getting fully charged, the capacitor will again discharge sending current through the inductor in opposite direction (as plates have opposite polarities) and therefore causing magnetic flux to link with inductor in the opposite direction.



As before, on the complete discharge of capacitor, the magnetic flux linked with the inductor starts winding up. As a result, the induced e.m.f. so produced charges the capacitor, such that polarity of the plates is opposite to that in the previous case i.e. the same as it was in the beginning

The process, then, repeats itself indefinitely and the electromagnetic oscillations are produced as shown in figure.



The oscillations are produced due to continuous conversion of energy in the form of electric field between the plates of the capacitor into magnetic field inside the inductor and vice versa. If the circuit does not have any resistance, no dissipation of energy will take place and the amplitude of oscillations produced will remain constant. Such oscillations are called undamped oscillations. However, due to finite resistance of the circuit, the amplitude of oscillations goes on decreasing. It is because; a small amount of electric energy is dissipated in the form of heat energy during each oscillation. In other words, in practice, the damped oscillations are produced.

The frequency of the LC-oscillations produced is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

5.2.7 Wattless Current

Average power is given by

$$P_{av} = E_{rms} I_{rms} cos\phi$$

The phase difference between E_{rms} *and* I_{rms} is ϕ. We can resolve I_{rms} into two components $I_{rms}cos\phi$ *and* $I_{rms}sin\phi$

Here the component $I_{rms}cos\phi$ contributes for power dissipation and the component $I_{rms}sin\phi$ does not dissipate power. Therefore, it is called Wattless current.

5.2.8 Electric Generator

An electric generator or dynamo is a device used to produce electrical energy at the expense of mechanical thermal energy.

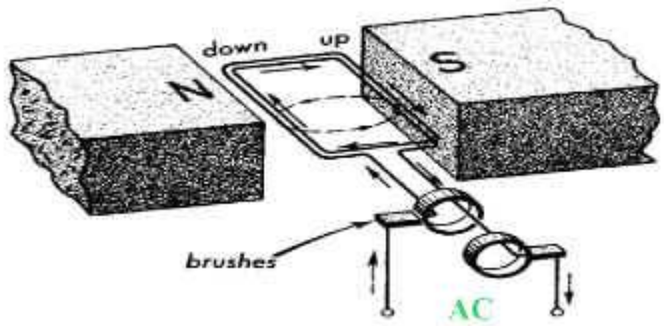
It words on the principal of electromagnetic induction, when a coil is rotated in a uniform magnetic field an induced emf is produced between its ends. The induced emf is given by e = $e_0$sin ωt. The direction of induced emf is alternating in nature.

5.2.8.1 AC Generator

Principle: A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction.

Alternating voltage may be generated by rotating a coil in the magnetic field or by rotating a magnetic field within a stationary coil. The value of the voltage generated depends on-

1. The number of turns in the coil.
2. Strength of the field.
3. The speed at which the coil or magnetic field rotates.

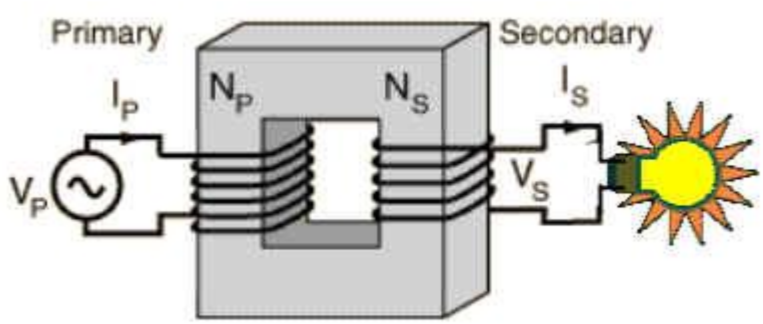


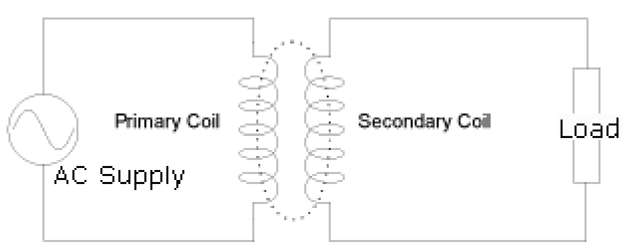
Working: Consider a rectangular coil having N turns and rotating in a uniform magnetic field with an angular velocity of ω radian/second. Maximum flux $Ø_m$ is linked with the coil when its plane coincides with the X-axis. In time t seconds, this coil rotates through an angle θ= ωt. In this deflected position, the component of the flux which is perpendicular to the plane of the coil is $Ø= Ø_m \cos ωt$. Hence flux linkage at any time are $NØ=NØ_m \cos ωt$.

According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux linkage of the coil.

5.2.9 Transformer

It is a device which works in AC circuits only and is based on the principle of mutual induction.





In its simplest form it consist of, two inductive coils which are electrically separated but magnetically linked through a path of low reluctance. If one coil (primary) is connected to source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction. If the second coil (secondary) circuit is closed, a current flow in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil.

Transformer is used to suitably increase or reduce the voltage in an AC circuit. Transformer which transforms strong AC at low voltage into a weaker current at high alternating voltage is called step up transformer. A step down transformer convert a weak current at high alternating voltage into a stronger current at lower alternating voltage.

For an ideal transformer

$$\frac{e_s}{e_p} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = k$$

Where k is known as transformation ratio.

For a step up transformer k > 1 but for a step down transformer k<1.

In a transformer input emf and output emf differ in phase by π radian.

The efficiency of a transformer is given by

$$\eta = \frac{output\ power}{input\ power} = \frac{V_S.I_S}{V_P.I_P}$$

For an ideal transformer η=100% or 1. However, for practical transformer $\eta \approx$ 85-90%

Possible causes due to energy loss in transformer are:

(i) heating due to winding resistance,
(ii) eddy current losses,
(iii) magnetic flux leakage,
(iv) Hysteresis loss.

To minimize these losses the transformer core is made as a laminated soft iron core.

Example: In an AC circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) P = zero
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2} E_0 I_0$

Solution: (b) For given circuit current is lagging the voltage by $\frac{\pi}{2}$ so circuit is purely Inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

Example: In a series resonant L-C-R circuit, the voltage across R is 100 V and R = 1 kΩ with C = 2μF. The resonant frequency ω is 200 rads^{-1}. At resonance the voltage across L is

(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

Solution: At resonance $\omega L = \frac{1}{\omega C}$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1\ A$$

So, voltage across L is given by

$$V_L = 1\,X_L = I\omega L$$

But $\omega L = \frac{1}{\omega C}$

$\therefore \quad V_L = \frac{1}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250\ volt$

5.2.10 Induction Coil

An induction coil is a type of electrical transformer that uses a low-voltage DC supply to produce high-voltage pulses. The coil is made up of two coils made up of insulating copper wire wound around a common iron core. The first coil is small, usually made up of tens or hundreds of turns of coarse wire. This is known as the primary winding. The second coil, known as the secondary winding, is made up of thousands of turns of fine wire.

The coil works by sending a small electric current through the primary coil. This creates a magnetic field. Since both primary and secondary windings are wrapped around a common iron core, the primary winding is coupled with the secondary winding. When the voltage going into the primary is suddenly interrupted or stopped, the magnetic field collapses quickly.
The sudden collapse of the magnetic field creates a high voltage pulse to be developed across the terminals connected to the secondary windings, by a process known as electromagnetic induction. Since the secondary coil has thousands of turns of wire, the pulse created is in thousands of volts. In most cases, when the pulse is created in the secondary coil, it creates a spark or jump between terminals. This is why the induction coil is often referred to as the "spark coil."

In order to get the induction coil to work, the electrical current that is fed to the primary coil has to be interrupted on a continuous basis. The device that causes the interruption is known as an interrupter. The interrupter that connects and breaks the current to the primary winding is made of a vibrating mechanical contact.

When a magnetic field is created in the primary winding, this field attracts or pulls an iron armature that is attached to a spring. This pulling of the iron armature breaks a pair of contacts that connect to a power source. Once the power source is broken, the magnetic field collapses, resulting in the spring closing the contacts. When the contacts close, the cycle starts over again.

Induction coils are used quite often in television sets and other electronic devices where low-voltage needs to be converted to high-voltage. A car's ignition system uses an induction coil to convert power from the battery. Induction heating is the use of an induction coil to provide localized, controlled heat objects placed inside the coil.

5.2.11 Electric Motor

Electric motors are everywhere! In your house, almost every mechanical movement that you see around you is caused by an AC (alternating current) or DC (direct current) electric motor.

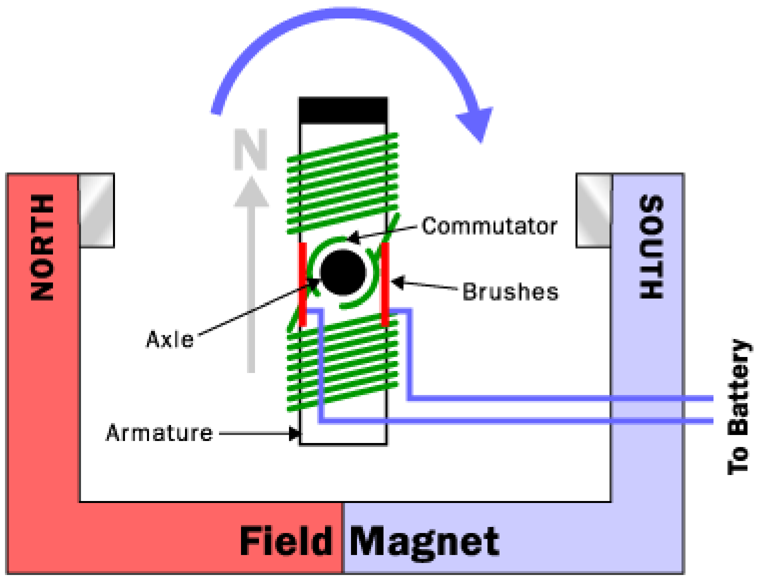
A simple motor has six parts:

1. **Armature or rotor**

2. **Commutator**
3. **Brushes**
4. **Axle**
5. **Field magnet**
6. **DC power supply of some sort**

An electric motor is all about magnets and magnetism: A motor uses **magnets** to create motion. Inside an electric motor, the attracting and repelling forces create **rotational motion**.

An electromagnet is the basis of an electric motor.



Parts of an electric motor.

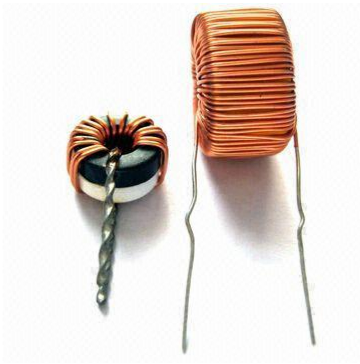
In the above diagram, you can see two magnets in the motor: The armature (or rotor) is an electromagnet, while the field magnet is a permanent magnet (the field magnet could be an electromagnet as well, but in most small motors it isn't in order to save power).

5.2.12 Choke Coil

A choke coil is an electrical appliance used for controlling current in an a.c. circuit without wasting electrical energy in the form of heat.

Principle: When AC flows through an inductor, the current lags behind the emf by a phase angle п/2.

Construction: A choke coil is basically an inductance. It consists of large number of turns of insulted copper wire wound over soft iron core. In order to minimize loss of electrical energy due to production of eddy currents, a laminated iron core is used.



Working: The choke coil is connected in series with the electrical device which operates on a low value of current. The inductive reactance of choke coil decreases the current. The average power consumed by the choke coil is

$P_{av} = E_v I_v \cos\pi/2 = 0$ (Since phase angle is $\pi/2$)

However, a practical inductance possesses a small resistance. Therefore, a practical inductance will consume a small average power given by

$$P_{av} = E_v I_v \times \frac{r}{\sqrt{r^2 + \omega^2 L^2}}$$

Here, $\frac{r}{\sqrt{r^2 + \omega^2 L^2}}$ is power factor ($\cos\varphi$) for practical inductance, which is series combination of L and r.

5.3 SOLVED EXAMPLES

Example 1: A coil having n turns and resistance RΩ is connected with a galvanometer of resistance 4RΩ. This combination is moved in time t second from a magnetic field B_1 weber to B_2 weber. What is the value of induced current in the circuit?

Solution: In a coil having n turns, the induced e.m.f. produced,

$$e = -n\frac{d\Phi}{dt}$$

Here, $d\phi = B_2 - B_1$; $dt = t$

$$\therefore \quad e = -\frac{n(B_2 - B_1)}{t}$$

The net resistance of the circuit,
R′=R+4R=5R
Therefore, induced current in the circuit,

$$I = \frac{e}{R'} = -\frac{n(B_2 - B_1)}{5\,R\,t}$$

Example 2: A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad s^{-1}. If the horizontal component of the earth's magnetic field is 0.2×10^{-4} T, then find the e.m.f. developed between the two ends of the conductor?

Solution: The induced e.m.f. produced across the two ends of the conductor,

$$e = \frac{1}{2}B\,l^2\ \ \omega = \frac{1}{2}\times 0.2\times 10^{-4}\times 1^2\times 5$$

$$=5\times 10^{-5}\ \text{V}=50\ \mu\text{V}$$

Example 3: When the current changes from +2A to -2A in 0.05 s, an e.m.f. of 8 V is induced in the coil. Find the value of the coefficient of self induction of the coil?

Solution: Here, dI = (-2)-2=-4A; dt=0.05s and e=8V

Now, $e = -L\,\frac{dI}{dt}$

Or $L = -\frac{e}{dI/dt} = -\frac{8}{-4/0.05} = 0.1\ H$

Example 4: The phase difference between the alternating current and e.m.f. is п/2. Which of the following cannot be the constituent of the circuit?

(A) C alone (B) L alone
(C) L, C (D) R, L

Solution: When the a.c. circuit contains L alone or C alone, the phase difference between the alternating current and e.m.f. is п /2.
When the a.c. circuit contains the combination of L and C, the phase difference is п.
When the a.c. circuit contains the combination of L and R, the phase difference is between 0 and п/2.

Example 5: In an LCR-series a.c. circuit, the voltage across each of the components. L, C and R is 50 V. What will be the voltage across the LC-combination?

Solution: The voltage across L and C are out of phase with each other.
Hence, the voltage across LC-combination is zero.

Example 6: In a transformer, number of truns in the primary is 140 and that in the secondary is 280. If current in primary is 4 A, then what is the current in the secondary?

Solution: Here, $N_p = 140$; $N_s = 280$; $I_p = 4$ A

$\therefore \quad I_s = I_p \times \frac{N_p}{N_s} = 4 \times \frac{140}{280} = 2\ A$