TRIGONOMETRY

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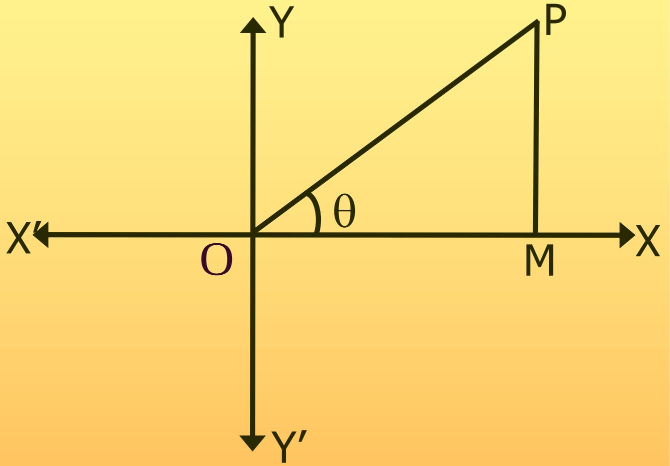
- **Trigonometric Functions:** These are the functions of angles defined in terms (or ratios) of slides of a right triangle.
  Let OX be an initial line. Suppose that it traces an angle θ, i.e., ∠XOP = θ in any of the four quadrants.



From P, drop a perpendicular PM on OX.

Then trigonometric functions (or circular functions) are defined as

$$\sin\theta = \frac{MP}{OP};\ \text{cosec}\ \theta = \frac{OP}{MP};\ \cos\theta = \frac{OM}{OP};\ \sec\theta = \frac{OP}{OM};\ \tan\theta = \frac{MP}{OM};\ \cot\theta = \frac{OP}{MP};$$

- **Relations between trigonometric Ratios:**

✓ Cosec $\theta = \frac{1}{\sin\theta}$ or cosec $\theta \times$ sin θ = 1

✓ sec $\theta = \frac{1}{\cos\theta}$ or sec $\theta \times$ cos θ = 1

✓ cot $\theta = \frac{1}{\tan\theta}$ or cot $\theta \times$ tan θ = 1

✓ $\sin^2\theta + \cos^2\theta = 1$

✓ $\sec^2\theta = 1 + \tan^2\theta$.

✓ $\text{cosec}^2\theta = 1 + \cot^2\theta$.

- **T-ratios of Sum or Difference of Two Angles:** If α, β are measures of two angles, then

✓ $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$.

✓ $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$.

✓ $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$.

✓ $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$.

✓ $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$ ✓ $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$

Hold good if none of the angles α, β, $\alpha + \beta$, $\alpha - \beta$ is an odd multiple of $\pi/2$.

- ✓ $\cot(\alpha + \beta) = \dfrac{\cot\alpha \cot\beta - 1}{\cot\alpha + \cot\beta}$
- ✓ $\cot(\alpha - \beta) = \dfrac{\cot\alpha \cot\beta + 1}{\cot\alpha - \cot\beta}$

- **Formulae which express the Sum or Difference into Product**

- ✓ $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
- ✓ $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$
- ✓ $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$
- ✓ $\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

- **Formulae which express Products as Sum or Difference of Sines and cosines:**

- ✓ $2\sin A \cos B = \sin (A + B) + \sin (A - B)$
- ✓ $2 \cos A \sin B = \sin (A + B) - \sin (A - B)$
- ✓ $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$
- ✓ $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$
- ✓ $\sin (A + B) \sin (A - B) = \sin^2 A - \sin^2 B$
- ✓ $\cos (A + B) \cos (A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

- **T-ratios of Multiple Angles**

✓ $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

✓ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

✓ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

- **T-ratios of 3θ in terms of those of θ**

✓ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

✓ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

✓ $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

- **T-ratios of Sub-multiple Angles**

✓ $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{2\tan\theta/2}{1+\tan^2\theta/2}$

✓ $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 1 - 2\sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1$

$$= \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$$

✓ $\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2}$

- **Inverse Trigonometric Functions:**

✓ **$y = \sin^{-1} x$:** It is defined as the angle y, which lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, whose sine is x i.e., sin y = x

Similarly, we may define other inverse trigonometric functions.

- **Trigonometric equations:** An equation containing trigonometric functions of unknown angles, is known as trigonometric equation.
- **Formulae**

✓ Solution of sin $\theta = 0$ is $\theta = n\pi, n \in I$

✓ Solution of cos $\theta = 0$ is $\theta = (2n + 1)\dfrac{\pi}{2}$, $n \in I$

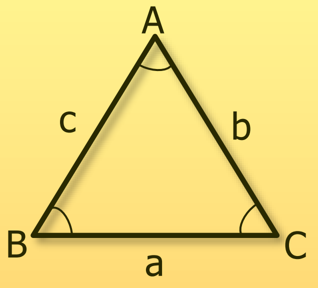
✓ Solution of tan $\theta = 0$ is $\theta = n\pi, n \in I$

✓ Solution of sin θ= sin α is $\theta = n\pi + (-1)^n \alpha, n \in I$
or $\theta = 2n\pi + \alpha n \in I$ or $\theta = (2n + 1)\pi - \alpha, n \in I$

✓ Solution of cos θ = cos α is $\theta = 2n\pi \pm \alpha, n \in I$

✓ Solution of tan θ = tan α is $\theta = n\pi + \alpha, n \in I$

- **Solution of triangles:** The process of obtaining the unknown elements from the given ones is called the solution of a triangle.



✓ In any triangle, sides and proportional to the sines of the opposite angles. That is, in a triangle ABC

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

(This formula is also called the *Law of Sines* or the *Sine Formula*)

✓ If A, B and C are angles of a triangle and if a, b and c are lengths of the sides opposite to A, B and C, respectively, then

$a = b \cos C + c \cos B$, $b = c \cos A + a \cos C$,

$c = a \cos B + b \cos A$

(These are known as Projection Formulae.)

- If A, B and C are angles of a triangle and if a, b and c are lengths of the sides opposite to A, B and C, respectively, then
  $a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = c^2 + a^2 - 2ca \cos B$,
  $c^2 = a^2 + b^2 - 2ab \cos C$
  (These formulae are also known as the *Law of Cosines or the Cosine Formulae*)
- In any triangle ABC, if a + b + c = 2s then

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}, \sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

- In any triangle ABC, if a + b + c = 2s then

$$\cos\frac{A}{2} = \frac{\sqrt{s(s-a)}}{bc}, \cos\frac{B}{2} = \frac{\sqrt{s(s-b)}}{ca}, \cos\frac{C}{2} = \frac{\sqrt{s(s-c)}}{ab}$$

- ✓ In any triangle ABC, if a + b + c = 2s then

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}},\ \tan\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}},\ \tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

- ✓ The area 'Δ' of a triangle ABC is given by

$$\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C$$

- ✓ In triangle ABC,

$$\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2},\ \tan\frac{C-A}{2}=\frac{c-a}{c+a}\cot\frac{B}{2},\ \tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$$

Thank You…